Stand Development Model BEM

by Günter Wenk & Colleagues

$$h = A \cdot \left[1 - e^{-k \cdot t} \right]^{p}$$
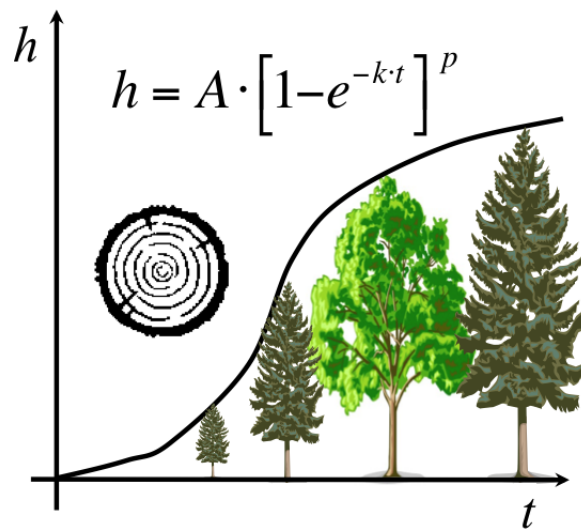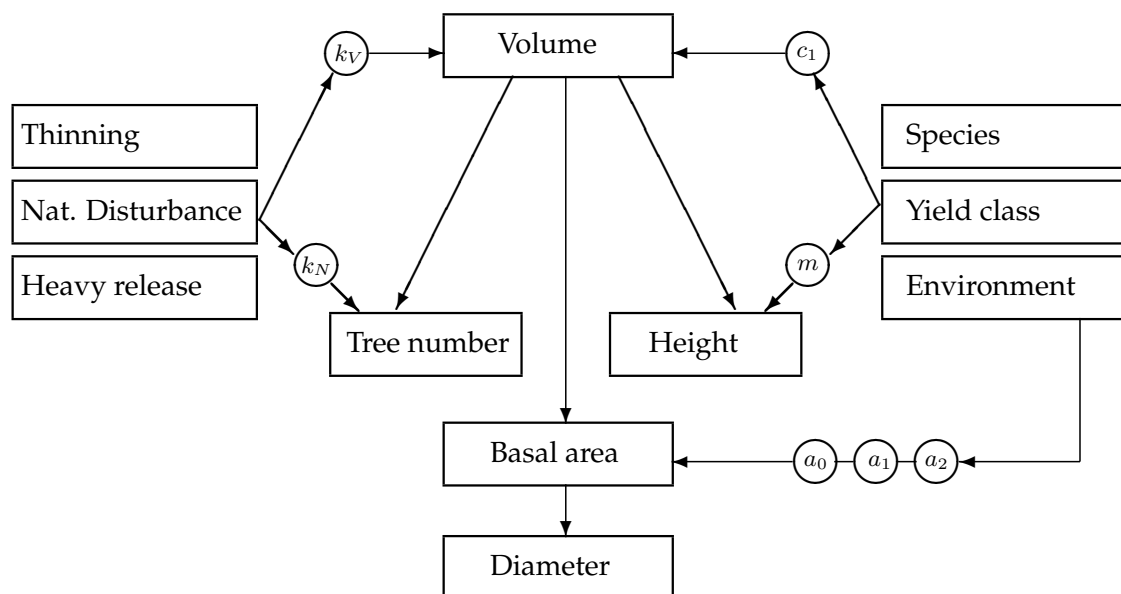
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February / March 2014

1 Introduction

The stand development model BEM is one of the legacies of the late Günter Wenk, professor in forest growth and yield science at Dresden Technical University, Tharandt campus, Germany. It is an intriguing and unique model approach, because it is based on the theory of relative growth as described in my leaflet "The Theory of Relative Increment" on the CRANCOD website.

After developing the theory of relative increment between 1969 and the 1990s, the stand development model was first programmed in the mid 1980s and thus made available to a larger audience.

The theory of relative increment as well as the foundations of BEM were extensively published in Wenk's textbook Wenk *et al.* (1990) (in German) and also in the paper Wenk (1994).

Fig. 1 gives a flowchart of BEM to explain the dependencies of the model parameters and the model hierarchy. The model parameters can be freely selected and can also vary over time.

Figure 1. Model hierarchy, dependencies and roles of the main model parameters within BEM (modified from Nicke, 1988).

According to Murphy and Pommerening (2010) Wenk's approach is unique because:

1. Relative increment is modelled rather than absolute increment.

2. The primary focus is on volume growth and its allometric relation to height and diameter growth. This modelling sequence effectively reduces error propagation.

3. The model is generic in that it can be applied at different scales.

The last property implies that the model is primarily based on mathematical rather than statistical relationships. Rather than deriving model parameters by pooling data from large geographic areas Wenk treated each stand as an individual case. As such the model parameters are input and output parameters at the same time: They also serve as interpretable characteristics summarising the development of a whole forest stand in just a few numbers.

Another advantage of the BEM is its flexibility to allow "backcasting", i.e. simulating historical stand conditions, which makes the model quite unique.

The biggest scientific challenge is to find a relationship between the growth parameter c_1 (see my leaflet "Relative Increment – Theory") and yield class (Murphy and Pommerening, 2010), so that yield tables for arbitrary yield classes can be produced by BEM.

The purpose of this leaflet is to present the basic algorithm of BEM in a consistent way and to provide an appropriate background for the corresponding R/C++ implementation `BEM.R`. It has always been Prof. Wenk's desire that his concepts and modelling approaches are made available to a wide audience of interested researchers and I hope that my R/C++ implementation together with this brief documentation will contribute to this.

2 Important abbreviations

The model and corresponding documentation uses a number of variables and they are here presented as abbreviations. For ease of understanding outside the German science community I have renamed Wenk's original abbreviations after the British yield tables with modifications.

dg	Quadratic mean diameter in cm.
CAI	Current annual increment in m^3 per hectare and year.
CVOL	Cumulative volume in m^3 per hectare of the present and all previous thinnings.
FH	Form height.
MAI	Mean annual increment, i.e. cumulative volume production to date divided by age in m^3 per hectare and year.
MBA	Mean stand basal area in m^2 per hectare before thinning.
Mh	Height growth multiplier.
MHT	Mean stand height in metre.
MTREES	Number of trees per hectare of the main stand before thinning.
Mv	Volume growth multiplier.
MVOL	Volume in m^3 per hectare of the main stand before thinning.
pH	Relative height increment.
pV	Relative volume increment.
pNN	Tree number removal rate.
pNV	Volume removal rate.
TTREES	Number of trees per hectare removed by thinning.
TVOL	Volume in m^3 per hectare removed by thinnings.
RTREES	Residual number of trees per hectare after thinning.
RVOL	Residual volume in m^3 per hectare after thinning.
TVP	Total volume production in m^3 per hectare.

As BEM is a generic model the user can define any units as long as this is consistent throughout the range of input variables. The outputs are then provided in the same units. Volume can, of course, also be replaced by biomass.

3   Basic projection algorithm

Also in this section abbreviations are used again instead of variables. They correspond with the headers of the output tables and with the names of the column vectors in R. The algorithm is also known as Tharandt algorithm and documented in Wenk *et al.* (1990, p. 284f.), Wenk and Nicke (1985), Nicke (1988) and Wenk (1994).

For a visual overview of the growth process please refer back to Fig. 1. The sum-

mary characteristics MTREES, RTREES, TTREES, MVOL, RVOL and TVOL are also explained in Fig. 2.

3.1 Forecasting growth

This section describes the procedure of forecasting from a given *reference age* to the *final age*.

In a first step, the volume of the trees to be thinned is calculated as the product of the volume of the main stand before thinning, MVOL, and the volume removal rate, pNV. Then the residual stand volume, RVOL, constitutes the difference between the main stand volume before thinning, MVOL, and the volume yield from thinning, TVOL.

$$\text{TVOL} = \text{MVOL} \times \text{pNV} \tag{1}$$

$$\text{RVOL}_{t_0} = \text{MVOL}_{t_0} - \text{TVOL}_{t_0} \tag{2}$$

$$\text{MVOL}_{t_1} = \text{RVOL}_{t_0} \times \text{Mv}_{t_1} \tag{3}$$

The main stand volume before thinning, MVOL, for the next time step, t_1, is calculated as the product of the residual volume of the previous time step, RVOL_{t_0}, and the volume growth multiplier, Mv_{t_1}.

In analogy to volume projection, the development of trees per hectare is modelled.

$$\text{TTREES} = \text{MTREES} \times \text{pNN} \tag{4}$$

$$\text{RTREES}_{t_0} = \text{MTREES}_{t_0} - \text{TTREES}_{t_0} \tag{5}$$

$$\text{MTREES}_{t_1} = \text{RTREES}_{t_0} \tag{6}$$

The only difference is that RTREES_{t_0} also constitutes MTREES_{t_1} and that there is no multiplier involved.

For a visual explanation of the summary characteristics MTREES, RTREES, TTREES, MVOL, RVOL and TVOL see also Fig. 2.

Mean stand height is calculated using the height growth multiplier, Mh. Mh is derived from pV and the allometric coefficient, m, see Section 4.3.

$$\text{MHT}_{t_1} = \text{MHT}_{t_0} \times \text{Mh}_{t_1} \tag{7}$$

3.2 Backcasting growth

This section describes the procedure of backcasting from a given *reference age* back to the *initial age*. If reference age and initial age are the same, this part of the model code is skipped.

In a first step, residual stand volume is calculated by dividing the volume of the main stand before thinning at reference age by the volume growth multiplier. Then volume of the main stand before thinning at the next lower age is obtained from dividing residual stand volume by the complement of the volume removal rate.

$$\text{RVOL}_{t_0} = \text{MVOL}_{t_1}/\text{Mv}_{t_1} \tag{8}$$

$$\text{MVOL}_{t_0} = \text{RVOL}_{t_0}/(1 - \text{pNV}) \tag{9}$$

In analogy to the last step, the characteristic trees per hectare of the main stand is backcasted.

$$\text{MTREES}_{t_0} = \text{RTREES}_{t_0}/(1 - \text{pNN}) \tag{10}$$

Finally, mean stand height is calculated backwards by dividing by the height growth multiplier, Mh.

$$\text{MHT}_{t_0} = \text{MHT}_{t_1}/\text{Mh}_{t_1} \tag{11}$$

This simplified backcasting algorithm yields the necessary start values MVOL, MTREES and MHT at initial age. The forecasting algorithm of the previous section then continues to project the full simulation time frame.

3.3 Derived summary characteristics

Below is a list of computations for derived summary characteristics. Basal area, MBA, is calculated from stand volume and form height.

$$\text{MBA} = \frac{\text{MVOL}}{\text{FH}} \tag{12}$$

$$\text{dg} = 100 \cdot \sqrt{\frac{4 \cdot \text{MBA}}{\pi \cdot \text{MTREES}}} \tag{13}$$

$$\text{TVP} = \text{MVOL} + \text{CVOL} \tag{14}$$

$$\text{MAI} = \frac{\text{TVP}}{\text{Age}} \tag{15}$$

$$\text{CAI} = \text{MVOL} \times \frac{\text{pV}}{\text{Period length}} \tag{16}$$

The quadratic mean diameter, dg, is computed using stand basal area and trees per hectare. Total volume production, TVP, is the sum of stand volume and cumulative volume, CVOL, which is the sum of the present and all previous thinnings. The following two formulae give the definitions of the two stand increments.

4 Detailed formulae

A precondition for the growth multipliers and the removal rates is a function of relative increment. The function used in BEM is described in detail in my leaflet "Relative Increment – Theory" (Section 2).

4.1 Volume removal rate function

The volume removal rate function after Wenk/Nicke, pNV is linked to the function of relative volume increment, pV.

$$pNV = pV - c_0 \cdot pV^{k_V} \tag{17}$$

A safe assumption is to set $c_0 = 1$ and negative values simulate a decrease of standing volume.

An earlier, alternative removal rate function was developed by Zimmermann (1974). However, this function can only be applied to 10-years projection periods. Also, this

function does not allow to remove volume in access of the increment. The function is included in the code of BEM.R, but disenabled with a hash key (#).

$$pNV = pV - e^{-c_1 \cdot k_V \cdot t' \left(1 + a \cdot e^{-b \cdot t'}\right)} \tag{18}$$

As a default b can be set to 1.5. For parameter a values between 1 and 20 can be selected. The larger the value of a the more intensely young stands are thinned.

The parameter k_V defines thinning intensity and normally ranges between 1.2 and 1.6. A value of 1.2 defines a weak thinning, 1.4 a moderate and 1.6 a heavy thinning. Negative values of c_0 lead to a decrease of stand volume. Naturally removal rate functions can also be used for modelling mortality.

Other, simpler solutions include the ratio of pNV/pV, i.e. the volume removal of the time period to come is defined by a certain percentage of the volume increment of the previous period. This is based on the biologically plausible assumption that the volume removal of a future period is dependent on past growth. A value of 0.8 for example means that in the next period 80 % of the volume increment of the previous period will be cut. Also values of 150 % or 200 % are possible and the user can thus simulate a decrease of standing volume.

Another possibility is to use parameter Q, which is used to define the volume removal rate as follows:

$$pNV = 1 - \frac{Q}{M_v}, \tag{19}$$

where M_v is the volume multiplier. Q can be be interpreted as a volume stocking degree. $Q = 0.5$ implies that the standing volume will be reduced by 50 % while $Q = 0$ leads to a clearfelling.

These alternatives can also be used for simulating disturbances. They have not been implemented in the R code yet but can easily be added.

4.2 Tree number removal rate function

The tree number removal rate is related to the volume removal rate by the tree number removal factor, k_N, which incidentally corresponds to the ratio of the mean volume of thinned trees and mean volume of the main stand before thinning, also referred to as Magin's k factor.

$$pNN = \frac{pNV}{k_N} \tag{20}$$

The tree number removal rate is inversely proportional to the volume removal rate and $1/k_N$ is the proportionality factor. k_N describes the thinning type and $k_N < 1$ indicates a low thinning (= thinning from below), $k_N = 1$ an indifferent thinning and $k_N > 1$ a crown thinning.

4.3 Relative height increment

Relative stand height is simply derived from relative volume through the allometric coefficient m.

$$pH = 1 - e^{\frac{1}{m \cdot ln 1 - pV}} \tag{21}$$

ph can then be turned into the height growth multiplier using Eq. (5) in the leaflet "Relative Increment – Theory".

4.4 Form height relationship

The form height relationship used in the implementation `BEM.R` is based on the following formula:

$$\text{FH} = a_0 + a_1 \cdot \text{MHT} + a_2 \cdot \text{MHT}^2 \tag{22}$$

However, other functions are also possible. For suitable parameter values see Table 3 in the Appendix.

5 Model input

5.1 Initial stand data

BEM requires information on MVOL, MTREES and MHT. Also a vector (`Age`) containing the *reference* and the *final age* (`fAge`) plus a range of assessment ages (for which an output is desired) must be provided. These age values define the projection time steps. The *reference age* (`rAge`) is the age, which corresponds to the values of MVOL, MTREES and MHT specified by the user.

In addition the model needs information on the initial age (`iAge`) to assess, whether backcasting is required.

In the model implementation `BEM.R`, the age range is specified as a vector using the sequence function `seq()`, e.g. `seq(40, 160, interval)`. Here the first number, 40,

corresponds to the reference age and the second number, 160, to the final age. An initial age differing from the reference age can be specified in the next line, e.g. `iAge <- 20`, which in this case activates the backcasting routine.

5.2   Model parameters

The user needs to provide the parameters of the relative volume increment function c_1, c_2, c_3 and c_4. In most cases c_3 can safely be set to a value of 0.4 and $c_4 = 0$. The parameter c_4 can be used for modelling environmental change. Experience has also shown that values of $c_4 = -0.05$ or $c_4 = -0.1$ are good for modelling oak growth. Suggestions for c_1 and c_2 and how to sample them in a given stand can again be found in my leaflet "Relative Increment – Theory", in Murphy and Pommerening (2010) and in Table 1 of the Appendix.

Then the thinning parameters k_V and k_N (see Table 2 of the Appendix) as well as the allometric coefficient (see Table 1) and the c_0 parameter of the volume removal rate function are required.

Finally the user has to specify the parameters a_0, a_1 and a_2 of the form height relationship. Default values of the model parameters can be found in Table 3 in the Appendix of this document.

For temperate forests the user should select an age transformation of 10 for the relative increment function, tropical trees species often require a smaller value.

Under normal conditions the growth parameter c_1 does not change during the lifetime of a forest stand. However, pollution or climate change can lead to changing growth patterns. In a similar way forest management can change over the years. To take this into account it is possible to define variable model parameters c_1, m, k_V, k_N and c_0. Constant model parameters you can specify in `BEM.R` by using the replicate function `rep()`, e.g. `c1 <- rep(0.19, 10)`. In this case a vector is created with 10 values of 0.19. For varying model parameters the user simply provides the whole vector, e.g. `c1 <- c(0.33, 0.33, 0.33, 0.28, 0.26)`.

6   Model output

Output tables are printed in the R console. In addition example code for producing output graphs like in Fig. 2 is also given.

In addition there is also a possibility of producing a LaTeX/pdf report of the model output by using the `Sweave` package (see `www.stat.uni-muenchen.de/~leisch/Sweave/`). The report syntax has already been programmed in the file `BEMreport.Rnw`. This file needs to be copied into the folder where you have saved `Bem.R` and the file path also required adjustment. `BEMreport.Rnw` uses the data frames `inputVector`, `vgrowth Data`, `ngrowthdata`, `rgrowthData`, `xx` and `yy`.
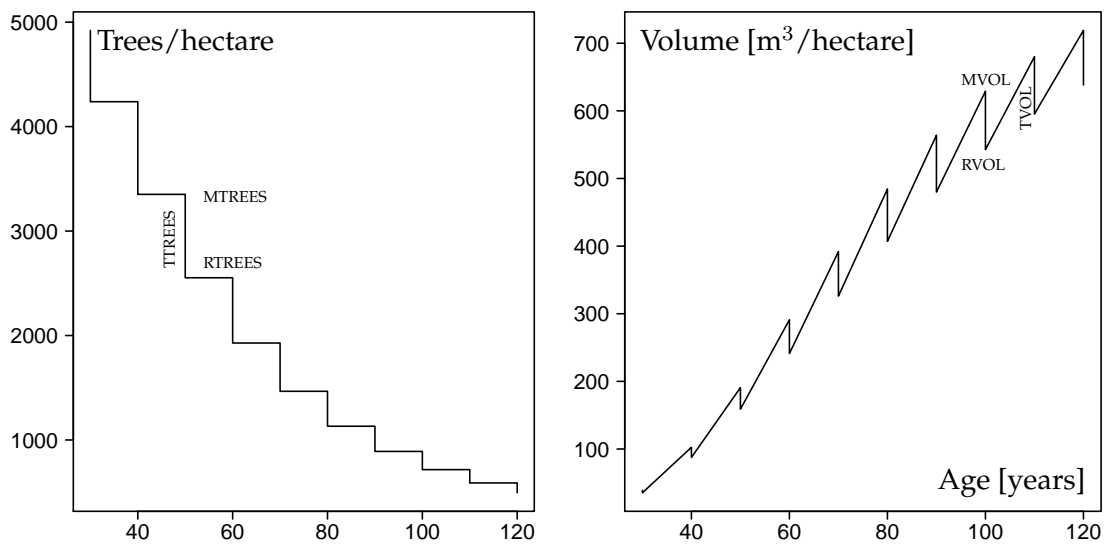
Figure 2. Trees/hectare and volume/hectare as output examples of Wenk's BEM model. The graphs also explain the meaning of the summary characteristics MTREES, RTREES and TTREES (left) and MVOL, RVOL and TVOL (right), respectively.

7   Final remarks

BEM is also capable of handling mixed and multi-structured forests. In addition, the model can project or backcast groups or cohorts of trees like for example frame trees. For this the current implementation of the model needs to be extended.

VSLM

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8   Appendix: Default model parameter values

Table 1. Growth parameters depending on tree species and yield class.

| Tree species | c_1 for yield class | | | | | c_2 | c_3 | c_4 | m |
	very high	high	average	low	very low				
Betula spp.	0.33	0.33	0.33	0.33	0.33	1.0	1.0	0.00	2.8
Fagus spp.	0.19	0.18	0.17	0.16	0.15	1.0	0.4	0.00	2.9
Pseudotsuga spp.	0.26	0.25	0.24	0.23	0.22	1.0	0.4	0.00	2.5
Quercus spp.	0.20	0.19	0.18	0.17	0.16	1.0	1.0	-0.05	3.1
Alnus spp.	0.29	0.29	0.29	0.29	0.29	1.0	1.0	0.00	3.5
Picea spp.	0.25	0.23	0.21	0.19	0.17	1.0	0.4	0.00	2.7
Pinus spp.	0.27	0.25	0.23	0.21	0.19	1.0	1.0	0.00	2.9
Larix spp.	0.22	0.21	0.20	0.19	0.18	1.0	1.0	0.00	2.8
Abies spp.	0.20	0.19	0.16	0.14	0.12	1.0	0.4	0.00	2.5

Table 2. Thinning intensity and thinning type.

Thinning intensity	k_V
Natural mortality	1.03
Weak low thinning	1.25
Moderate low thinning	1.45
Heavy thinning	1.65
Combined thinning	
Age < 50 years	1.75
$50 \leq$ age < 80 years	1.45
Age ≥ 80 years	1.20

Thinning type	k_N
Low thinning I	0.45
Low thinning II	0.55
Low thinning III	0.65
Variable frame tree thinning I	0.75
Variable frame tree thinning II	0.85
Schematic thinning	1.00
Crown thinning I	1.10
Crown thinning II	1.30

Table 3. Form height parameters based on different volume types, i.e. 1 – total stem volume, 2 – volume of tree parts with a diameter of 7 cm and above, 3 – all aboveground tree volume = 2 + branch volume.

Tree species	Form height parameters			Volume type	Source
	a_0	a_1	a_2		
Fagus spp.	0.50000	0.50000	-0.00180	1	Wenk unpublished
Fagus spp.	-4.08447	0.75980	-0.00382	2	
Fagus spp.	-2.73700	0.60100	0.00006	2	Kramer (1982)
Fagus spp.	-1.04112	0.61042	-0.00368	1	Gerold unpublished
Quercus spp.	-7.03600	1.12400	0.01200	2	
Quercus spp.	0.14790	0.47618	0.00011	1	Wenk unpublished
Alnus spp.	0.40000	0.45000	0.00050	1	Wenk unpublished
Picea spp.	1.29000	0.49300	-0.00300	1	Gerold (1984)
Picea spp.	0.45400	0.46600	-0.00200	1	Gerold unpublished
Picea spp.	0.50000	0.49300	-0.00300	1	
Picea spp.	-2.90000	0.87000	-0.01000	2	Krenn (1964)
Picea spp.	-4.25800	0.94000	-0.01100	2	Kramer (1982)
Pinus spp.	1.37000	0.41000	-0.00040	1	Wenk unpublished
Pinus spp.	1.50400	0.37950	0.00090	1	Wenk unpublished
Pinus spp.	0.64800	0.47800	0.00200	1	Teshome (1987)
Pinus spp.	-1.64100	0.60100	0.00300	3	Kramer (1982)
Larix spp.	0.50000	0.45000	0.00005	1	Wenk unpublished
Larix spp.	1.50000	0.50000	-0.00180	1	Wenk unpublished
Abies spp.	1.30000	0.50000	-0.00150	1	Wenk unpublished
Betula spp.	0.50000	0.40000	-0.00100	1	Wenk unpublished
Pseudotsuga spp.	0.50000	0.47000	-0.00150	1	Wenk unpublished