

1 Basic tree variables, forestry summary characteristics and biodiversity measures

English: forest mensuration; *Welsh:* mesuriad coed; *German:* Waldmesslehre, Dendrometrie; *Russian:* лесная таксация, дендрометрия; *Spanish:* dendrometría; *French:* dendrométrie.

Forest mensuration concentrates primarily on the quantitative assessment of trees and forest stands at a given point in time during their lives and provides the data required for efficient forest management (Laar and Akça, 1997).

Forest mensuration is a branch of biometrics dealing with the methods and conventions of measuring and processing tree and forest data. Forest mensuration comprises

- measurement and analysis of important individual tree and forest stand variables, e.g. diameter at breast height (DBH), height, basal area etc. including measures of structural diversity (sections 1.1 and 1.2),
- tree and forest growth dynamics,
- sampling and monitoring,
- basics of forest growth models.

The purpose of section 1.1 is to give a brief introduction to quantitative aspects of silviculture and can therefore only be forest mensuration in a nutshell. Silvicultural decisions are based on individual tree and forest stand characteristics. They are required for interpretation and the formulation of targets. Silvicultural literature assumes a good understanding of these basic statistics.

There are various ways to express structural diversity quantitatively and the most basic ones are also introduced in this section. Any science dealing with living beings must inevitably reflect the principles of growth dynamics. Sampling and monitoring are crucial to silviculture and forest management since they provide information on the response of trees to human interventions and other impacts.

Modern forest growth models are not only tools to produce production forecasts but can also be employed as decision support systems which can be used to simulate tree growth, human interventions and other biotic and abiotic impacts. Forest growth models can inform about the likely success of woodland management scenarios and the change in biodiversity as a result of this. Used as simulators they put silviculturists into the position of anticipating the lifetime of several tree generations.

Figure 1.1 illustrates the essential role of forest mensuration in practical silviculture. Data measured and collected in the field are rarely in the format required for the planning and decision making process and computational data processing is, therefore, a key task in silviculture (Puumalainen *et al.*, 1998).

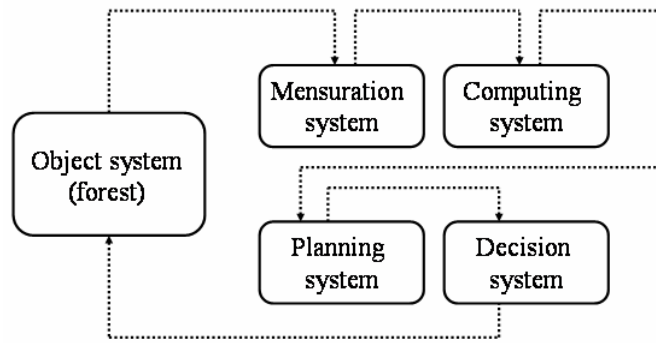


Figure 1.1. The computing process within the context of silviculture. Modified from Puumalainen et al. (1998).

Because of lack of space instruments which can be employed to measure tree and forest stand variables cannot be described here. The same applies to the quantification of estimation errors, which is an essential part of data computation (Krebs, 1999). The reader is referred to the literature (e.g. Laar and Akça, 1997; Krebs, 1999; Philip, 1994). Most variables are easy enough to compute in spreadsheet software such as MS EXCEL. The author of this book developed the CRANCOD software which can compute a wide variety of different tree variables including the more complex spatial indices. The CRANCOD software is downloadable on <http://tyfcoed.bangor.ac.uk>.

1.1 Important individual tree variables

In a statistical sense forest stands can be interpreted as *populations* and the individual trees that occur in the stand under study as *units* of the population. A population is an aggregate of *unit values*, where the unit is the object upon which the observation is made and the value is the observed aspect of that object. The complete set of values of a variable in a given situation is a population. A forest stand can for example serve as a study population in which the unit being observed is the individual tree, and the value being observed is the diameter at breast height (DBH) (Laar and Akça, 1997).

Figure 1.2 illustrates the key individual tree variables in silviculture.

English: age; *Welsh:* oedran; *German:* Alter; *Russian:* возраст; *Spanish:* edad; *French:* ???.

The *age* of a tree is defined as the period of time which has elapsed since germination. On trees which produce recognisable annual layers of wood, age can be determined by one of the following methods:

- By felling the trees, counting their annual number of rings at stump height and adding the estimated period of time the tree requires to reach stump height.

- By counting the number of rings on increment cores extracted at breast height or lower, with the aid of an increment borer and adding the estimated number of years required to reach the point of extraction.
- By counting the number of internodes (broadleaves) or whorls (conifers), provided they are clearly identifiable.

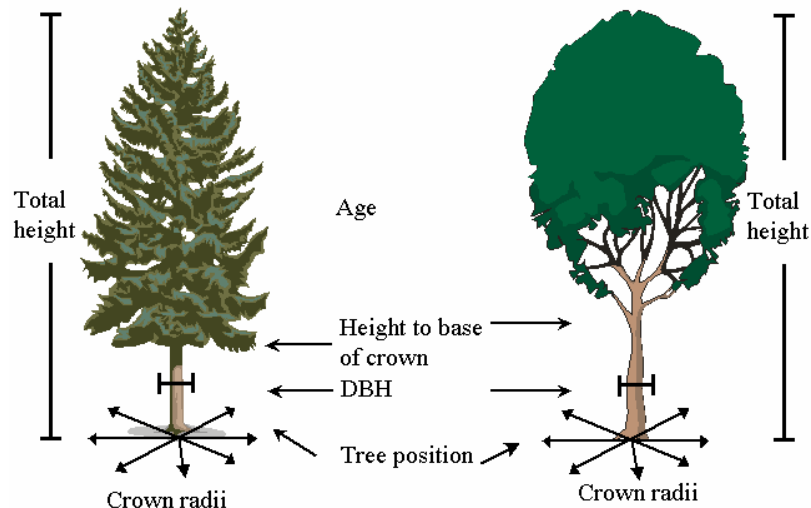


Figure 1.2. The main important tree variables in silviculture.

The advantages and disadvantages of the different methods are discussed in the relevant literature (Laar and Akça, 1997; Philip, 1994). Age is usually given in years and frequently used symbols include *age*, *a* and *t*.

English: diameter at breast height, breast height diameter (DBH); *Welsh*: diamedr ar uchder brest (DUB); *German*: Brusthöhendurchmesser (BHD); *Russian*: диаметр (дерева, ствола) на высоте груди, таксационный диаметр; *Spanish*: diámetro normal; *French*: diamètre à hauteur de poitrine, diamètre à 1,30 mètre.

In nursery studies, the diameter of a plant is always measured at its base and is described as root collar diameter. Tree diameter usually refers to the over bark diameter at a fixed distance from the base of the tree. In most countries, the point of measurement is located at 1.30m above ground level. The corresponding diameter is referred to as *diameter at breast height* or *breast height diameter* (DBH). In the literature the DBH is also often referred to as “diameter”. Common symbols are *DBH* and *d*. The diameter at breast height is usually measured in *cm* or *mm*. In practice, the breast height location is not always easy to determine and the literature gives guidelines for critical cases (see Laar and Akça, 1997).

English: basal area; *Welsh*: arwynebedd gwaelodol; *German*: Grundfläche, Kreisfläche; *Russian*: площадь (поперечного) сечения; *Spanish*: área basimétrica; *French*: surface terrière.

Derived from the diameter at breast height is the *basal area*. The basal area of a tree is defined as the cross-sectional area of a stem, usually measured at breast height. The

definition assumes the cross-sectional area of a stem to be of circular shape. Basal area can be either derived from the tree diameter (eq. 1.1) or from the circumference (eq. 1.2).

$$[1.1] \quad g_i = \pi \cdot r_i^2 = \pi \cdot \left(\frac{d_i}{2}\right)^2 = \frac{\pi}{4} \cdot d_i^2$$

where

- g_i basal area [m²] of tree i
 r_i radius [m] of the cross-sectional area of tree i
 d_i diameter [m] at breast height of tree i

$$[1.2] \quad g_i = \frac{c_i^2}{4\pi}$$

where

- c_i circumference [m] of tree i at breast height

Common symbols are g and BA . Basal area is usually measured in m^2 .

English: (total) tree height; *Welsh*: taldra coeden; *German*: Baumhöhe; *Russian*: высота дерева; *Spanish*: altura total del árbol; *French*: hauteur totale de l'arbre.

(Total) tree height is defined as the perpendicular distance between the top and base of a tree. In the case of perfectly straight stems of exactly vertical trees, tree height and stem length are identical. Tree height is usually measured in m , a common symbol is h .

English: height diameter ratio, slenderness; *Welsh*: cymhareb uchder/diametr; *German*: h/d -Verhältnis, h/d -Wert, Schlankheitsgrad; *Russian*: стройность; *Spanish*: relación altura-diámetro; *French*: rapport h/d .

Derived from tree height and diameter is the *height diameter ratio* or *slenderness* (h/d ratio). The height diameter ratio is an expression of tree morphology which results from growth conditions, namely site conditions, competition and management (Mitchell, 2000). The more growing space a tree is granted, the longer its crown and the smaller its height diameter ratio. It plays an important role as indicator variable of individual tree stability of coniferous trees in terms of wind and snow hazard. It is defined in formula 1.3.

$$[1.3] \quad h/d_i = \frac{h_i}{d_i}$$

where

- h/d_i height diameter ratio of tree i

h_i (total) height [m] of tree i
 d_i breast height diameter [m] of tree i

The height diameter ratio has no unit. Burschel and Huss (1997) suggested a system of interpreting individual tree stability of coniferous trees (see Table 1.1). The more slender trees are, the more they are prone to wind and snow damage. Open grown trees (Hasenauer, 1997; Smith *et al.*, 1992), which have grown up in absence of competition from other trees throughout their life, by definition have developed the highest possible degree of individual tree stability. Forest grown trees usually have much larger height diameter ratios than open grown trees (see Figure 1.3). Slenderness is also a useful indicator for judging the urgency of thinning operations (Abetz and Klädtke, 2002; Pretzsch, 1996).

Table 1.1. Interpreting height diameter ratios (modified from Burschel and Huss, 1997).

h/d value	> 100	80 - 100	< 80	< 45
Degree of individual tree stability	very unstable	unstable	stable	open grown tree

English: open grown tree; *Welsh:* coeden tir agored; *German:* Solitär(baum); *Russian:* дерево, выросшее на открытой местности; *Spanish:* árbol en crecimiento libre; *French:* arbre solitaire, arbre isolé.

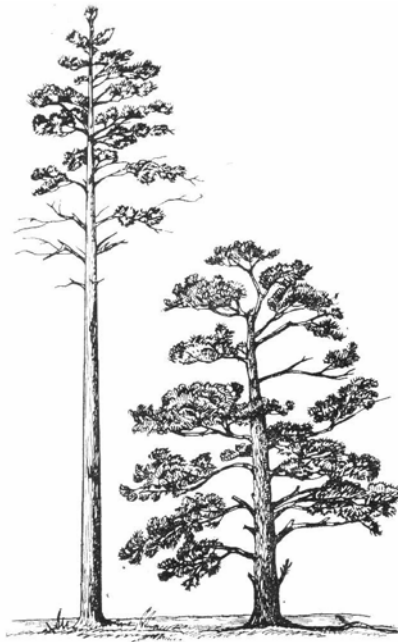


Figure 1.3. Comparison between the morphology of a forest tree (left) and an open-grown tree (right) of the same age for Scots pine. After Anučin (1982). H/d ratio and crown variables are able to quantify these morphological differences.

English: volume; *Welsh:* cyfaint; *German:* Volumen; *Russian:* объём (ствола); *Spanish:* volumen; *French:* volume.

Another individual tree variable derived from diameter and height is *volume*. Volume usually only refers to the volume of stems, frequently it is restricted to so-called *merchantable* or *commercial* volume, i.e. from the bottom up to a certain top diameter, e.g. 7cm. The exact volume of trees can only be determined by water displacement but this method is time consuming and only used in special circumstances. Tree volume is therefore estimated from diameter and height/length measurements. The basic idea is to express volume as a function of diameter and height or more precisely as a function of basal area and height. This comes from the idea that the stem of a tree can be understood as a cylinder. However, with increasing height towards the top there is an increasing deviation of a tree's stem from the shape of an ideal cylinder. This morphological characteristic of trees is referred to as *taper* and requires a correction factor introduced to the volume formula of a cylinder. This correction factor is referred to as the *form factor* and there are several methods of defining this (Laar and Akça, 1997; Philip, 1994).

English: taper, form factor; *Welsh*: tapr, ffactor ffurf; *German*: Abholzigkeit, Formzahl; *Russian*: сбег, сбежистость (ствола дерева), видовое число; *Spanish*: factor de forma, conicidad; *French*: décroissance, coefficient de forme.

Eq. 1.4 gives the full volume formula for a tree.

$$[1.4] \quad v_i = \frac{1}{4} \cdot d_i^2 \cdot \pi \cdot h_i \cdot f_i = g_i \cdot h_i \cdot f_i$$

where

h_i (total) height [m] of tree i
 d_i breast height diameter [m] of tree i
 f_i form factor of tree i

In recent years a number of form factor and volume functions have been developed to estimate volume directly from diameter at breast height and height. Tree volume is usually measured in m^3 , a common symbol is v .

English: crown; *Welsh*: brig(dyfiant); *German*: Krone; *Russian*: крона; *Spanish*: copa; *French*: couronne, houppier.



Figure 2.4. Crown variables.

Figure 1.4 illustrates the most important crown variables. The *crown radius* is the distance between the stem centre and the outer edge of the crown. Tree crowns can be very eccentric, for this reason the crown radius is usually determined as the arithmetic mean of 4-8 radius measurements in different directions. *Crown width* is crown radius multiplied by two. For *height to base of crown* there are various definitions in the literature. A sensible definition is “the first living branch (broadleaves), the first whorl with at least three living branches (conifers) from the base of the tree that is contiguous with the rest of the crown”.

Based on these crown variables a number of indices or ratios have been developed which play important roles in silvicultural research (see Table 1.2).

Table 1.2. Crown indices and ratios (Laar and Akça, 1997; Philip, 1994).

Eq. #	Name	Ratio	Other name
[2.5]	Crown ratio	$\frac{\text{crown length}}{\text{tree height}}$	<i>c/h</i> ratio
[2.6]	Crown form index	$\frac{\text{crown length}}{\text{crown width}}$	-
[2.7]	Linear crown index	$\frac{\text{crown width}}{DBH}$	<i>k/d</i> ratio
[2.8]	Crown spread ratio	$\frac{\text{crown width}}{\text{tree height}}$	-

The crown ratio is often used to assess individual tree stability of coniferous trees in a similar way as the *h/d* ratio, although the former is a better indicator of resistance to stem breakage while the latter indicates resistance to windthrow. An interpretation guide for the *c/h* ratio is given in Table 1.3.

Table 1.3. Interpreting crown ratios (modified from Schütz, 2001).

<i>c/h</i> value	≤ 0.30	$0.30 < c/h \leq 0.50$	> 0.50	≥ 0.62
Degree of individual tree stability	very unstable	unstable	stable	open grown tree

Both *c/h* and *h/d* ratios are often used to assess thinning requirements in managed forests (Pretzsch, 1996).

English: crown classes; *Welsh*: dosbarthiadau brigdyfiant; *German*: Baumklassen, Kraftsche (Baum, Stamm)Klassen; *Russian*: классы роста; *Spanish*: clases de copa, clases sociológicas; *French*: classes d'arbres, classes de grosseurs.

Classifying trees according to “social” or crown classes goes as far back as 1844 (Rozsnyay, 1979). Crown classes were originally devised for homogeneous even-aged

forests and have traditionally been used to define thinning types and intensities. Crown classes can be helpful tools to stratify the trees of a forest for special analyses. Trees are classified according to their total heights, i. e. their relative position in the forest stand, and their crown morphology. This visual assessment uses correlations between crown size and shape and competitive status of trees in a forest stand. For practical and scientific use it is also important to distinguish between light demanding and shade tolerant tree species. For example a mixed broadleaved forest stand with light demanding tree species in the overstorey can have overtopped beech, hornbeam and lime trees with well developed crowns (5a). There are different numbers of classes, names and coding systems in the literature (Köstler, 1956; Assmann, 1970; Burschel and Huss, 1997; Röhrig *et al.*, 2006; Smith *et al.*, 1997). Anglo-American sources often use letters (D, CD, SD, S) while numbers are common on the European continent (1-5). Figure 1.5 illustrates the system by Kraft.

1. Predominant trees:	Most dominant trees of a forest stand with exceptionally strongly developed crowns often above the level of the main canopy.
2. Dominant trees:	Dominant trees forming the main canopy and having comparatively well developed crowns.
3. Co-dominant trees:	Crowns extend into main canopy, but are comparatively weakly developed and narrow. Crowns start to degenerate. Lower limit of dominant trees.
4. Dominated trees:	Dominated trees with heavily squeezed or one-sided crowns (flag-shaped). a. Free crowns tops in the middle storey. b. Partly overtopped crowns in the understorey with the beginning of crown dieback.
5. Suppressed trees:	Crowns completely overtopped. a. but surviving (shade tolerant species only). b. dying or dead.

English: overstorey, middle storey, understorey; *Welsh:* troshaen, haen ganol, isdyfiant; *German:* Ober-, Mittelschicht-, Unterschicht (-stand); *Russian:* верхний ярус, средний ярус, нижний ярус; *Spanish:* piso superior, piso intermedio, subpiso; *French:* strate dominante, strate intermédiaire, strate inférieure.

Assmann (1970) associates classes 1-3 with the overstorey, class 4 with the middle and class 5 with the understorey. Assmann (1970) also suggested a quantitative definition of the three main canopy strata in a forest (see Table 1.4).

Table 1.4. The three main canopy strata according to Assmann (1970).

Canopy stratum	Height range in % of maximum stand height
Overstorey (= upper storey=)	> 80
Mid(dle) storey	50 – 80
Understorey (=lower storey)	< 50

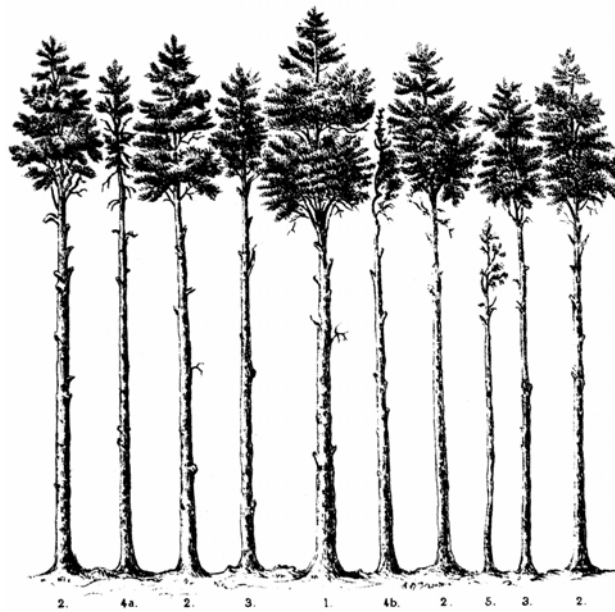


Figure 1.5. Crown classes according to Kraft (1884).

Often a more simplified system is used (Röhrig *et al.*, 2006). This has, of course, limitations in more complex woodlands involving tree species with very different light demands.

Dominant (D, 1):	Most dominant trees of a forest stand with crowns above the level of the main canopy facing hardly any lateral competition.
Co-dominant (CD, 2):	Dominant trees forming the main canopy with little lateral competition.
Sub-dominant (SD, 3):	Crowns extend into main canopy, but face strong lateral competition. Crowns are therefore smaller and irregular in shape.
Suppressed (S, 4):	Trees with overtopped crowns under the canopy of the main canopy.

Because of practical difficulties in deciding which trees in a forest stand belong to which crown class and the degree of subjectivity of judgements the original system by Kraft has been refined and modified various times. Kraft's system has often been criticised because timber quality and other tree characteristics are not reflected in it. An option to refine Kraft's original crown class system is to stratify trees according to a number of height zones and to classify the trees whose crown tips fall into one or another of the height strata separately for each stratum (Oliver and Larson, 1996).

Another tree stratification system developed from modern tending methods of selective thinnings as they are useful for the *frame tree management*. Rittershofer (1999) gives such a functional tree stratification system for commercial woodland management.

Frame trees (F)¹:	Trees selected for their outstanding vitality, stem quality, productivity, stability and crown morphology. Usually not more than 100-200 trees/ha.
Matrix trees (M):	All non-frame trees.
Hazard trees (H)²:	Diseased and damaged trees, trees potentially being a threat to frame trees:
Wolf trees (W):	Dominant, very competitive trees with poor stem quality.
Rubbing trees (R):	Sub-dominant and suppressed trees rubbing frame trees with larger branches or parts of their crown.
Whipping trees (Wh):	Sub- or co-dominant trees with slender stems and very small brush-like crowns. Whipping trees tend to move heavily in windy conditions and by doing so can seriously damage the crowns of their neighbours.
Competitors (C):	Neighbours of frame trees that potentially have a negative effect on the growth and development of frame trees.
Nurse trees (N):	Trees of the over-, mid- and understorey that are beneficial to frame trees, e.g. by providing shelter against climatic extremes, be preventing epicormic growth or the growth of invasive ground vegetation (bramble, bracken).
Indifferent trees (I):	Dominant or sub-dominant trees at sufficient distance to the frame trees and no effect on them. Such trees can potentially replace damaged or diseased frame trees.

A similar system is also given in Assmann (1970) where it is referred to as the *Danish tree classification*. Such systems are helpful for modern individual tree silviculture where trees need to be identified for thinning and release in the field. Quick decisions need to be made in such cases when marking trees for these two major groups. A classification system such as the one by Rittershofer may facilitate the decision making process. Figure 1.6 illustrates Rittershofer's frame tree based classification system.

¹ Köstler (1956) describes desirable tree characteristics in a commercial scenario as trees "with stems running right through with one definitely predominant axis. Such trees should be straight, erect and of circular cross-section. The crowns of frame trees should have a dominant leader. There should be a sufficient branch-free length of high-quality timber. The crown should be finely branched and should display a symmetrical structure, as with a symmetrical crown the stem grows concentrically."

² Other unfavourable tree characteristics in a commercial context include deeply forked stems, heavy and many branches, epicormic shoots, bent-over stems, cracks, spiral grain, broken leaders/tops, harvesting and extraction (bark abrasion) damage, insufficient root spread and anchorage in the soil, swellings, canker and other diseases.

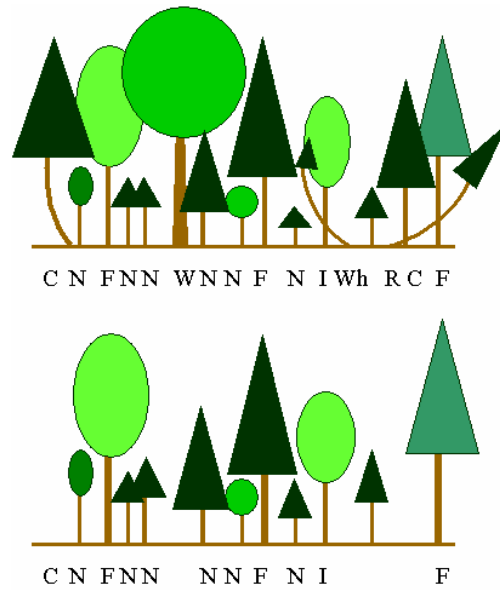


Figure 1.6. Crown classes according to Rittershofer (1999) and result of a selective thinning in favour of frame trees.

Simple modifications of this classification system could lead to an approach with primarily ecological objectives where valuable habitat and rare species trees are appointed as frame trees. A wide range of other tree classification systems has been developed (Röhrig *et al.*, 2006).

The following indices are measures to quantify the three aspects of structural tree diversity, *the diversity of tree positions*, *tree species diversity* and *the diversity of tree dimensions* (Pommerening, 2002; Aguirre *et al.*, 2003). For calculating these indices it is necessary to identify the nearest neighbours of the tree under study, generally termed the reference tree and because this gives a spatial dimension the following indices are referred to as being spatially explicit. The number of nearest neighbours, n , can be varied and needs to be determined according to the specific forest structure investigated (Pommerening, 2006), but often $n = 3$ and $n = 4$ are used (see Figure 1.7).

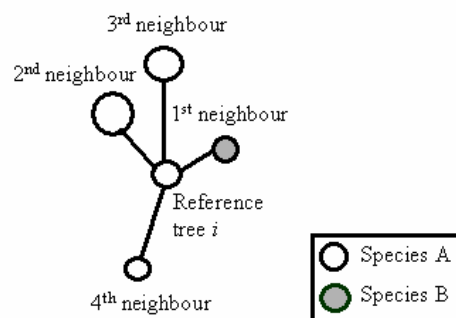


Figure 1.7. Example of a structural group involving 4 neighbour trees. This structural group of reference tree and neighbouring trees is the computational unit for the following indices (Pommerening, 2006).

Eq. 1.9 and 1.10 are indices of positional diversity. Eq. 1.9 is known as the *uniform angle index* (Gadow *et al.*, 1998; Hui and Gadow, 2002). α_j is the angle pointing away from the reference tree i to the neighbours j ($j = 1 \dots n$). If α_j exceeds 180° , then $\alpha_j = 360^\circ - \alpha_j$. Each angle α_j is compared to a standard angle α_0 calculated as $\alpha_0 = \frac{360^\circ}{n+1}$.

$$[1.9] \quad W_i = \frac{1}{n} \sum_{j=1}^n w_j \quad w_j = \begin{cases} 1, & \alpha_j < \alpha_0 \\ 0, & \text{otherwise} \end{cases} \quad W_i \in [0, 1]$$

If $\alpha_j < \alpha_0$ a value of 1 is allocated to the indicator variable w_j , otherwise 0. The indicator variable w_j is added up for all n neighbours. There are $n + 1$ possible values W_i can assume (see Figure 1.7). Low values of W_i correspond to a regular arrangement of neighbours around tree i , high values signify a clustered arrangement. Values around 0.5 reflect a random arrangement.

A similar index is the *mean directional index* (Corral-Rivas *et al.*, 2006), which calculates the length of the sum of unit vectors between a tree i and its nearest neighbours. α_{ij} is the angle subtended at the reference tree i by pairs of neighbour trees j and moving clockwise around the reference tree.

$$[1.10] \quad R_i = \sqrt{\left(\sum_{j=1}^n \cos \alpha_{ij} \right)^2 + \left(\sum_{j=1}^n \sin \alpha_{ij} \right)^2} \quad R_i \in [0, +\infty]$$

If the neighbouring trees of tree i are located in a perfect lattice square, $R_i = 0$. The more clustered the pattern of neighbours is the higher the R_i value. The expected value for a random configuration can be approximated with $E(R_i) = 0.5\sqrt{n \cdot \pi}$.

The mingling index (Füldner, 1995; Aguirre *et al.*, 2003) describes the species pattern around the reference tree. It belongs to the group of indices describing species diversity. M_i gives the proportion of the n nearest neighbours j of tree i which do not belong to the same species as tree i (see Figure 1.8).

$$[1.11] \quad M_i = \frac{1}{n} \sum_{j=1}^n m_j \quad m_j = \begin{cases} 1, & \text{species}_j \neq \text{species}_i \\ 0, & \text{otherwise} \end{cases} \quad M_i \in [0, 1]$$

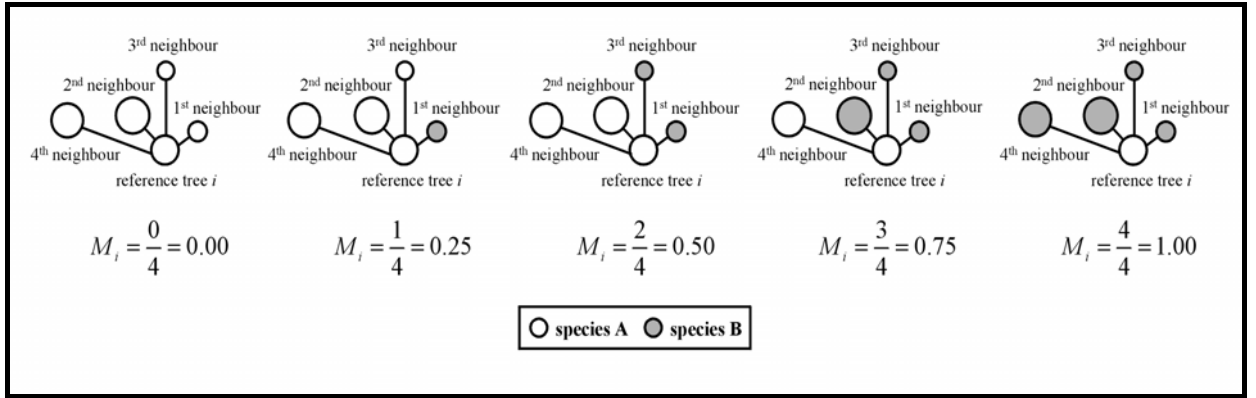


Figure 1.8. Illustration of the mingling index for $n=4$ neighbours. The neighbours are numbered according to increasing distance from the reference tree (Pommerening and Stoyan, 2006). The dominance and uniform angle index work in a similar way.

The larger the mingling variable M_i , the more the different tree species are intermingled. Small values indicate large groups of only one tree species and therefore segregation.

The following indices describe the diversity of tree dimensions. The *differentiation* index (Füldner, 1995; Pommerening, 2002) can be applied to any dimensional variable, e.g. to tree diameters and heights as in eq. 1.12 and compares the reference tree i to one of its neighbours j .

The value of TD_i or TH_i increases with increasing size difference between neighbouring trees. A value of 0 means that neighbouring trees have an equal size.

$$[1.12] \quad TD_i = 1 - \frac{\min(DBH_i, DBH_j)}{\max(DBH_i, DBH_j)} \quad TH_i = 1 - \frac{\min(h_i, h_j)}{\max(h_i, h_j)} \quad TD_i, TH_i \in [0, 1]$$

In a similar way as the differentiation the dominance index (Hui *et al.*, 1998; Aguirre *et al.*, 2003) can use any tree size variable, e.g. tree diameters or total heights. The indicator variable u_j is 1 if diameter or height of tree i exceeds that of the neighbouring tree j , otherwise 0.

$$[1.13] \quad U_i = \frac{1}{n} \sum_{j=1}^n u_j \quad u_j = \begin{cases} 1, & DBH_i, h_i > DBH_j, h_j \\ 0, & \text{otherwise} \end{cases} \quad U_i \in [0, 1]$$

Johann (1982) developed the *A-thinning index* (eq. 1.14) which defines a critical distance cd_{ij} between tree i and its n nearest neighbours depending on the thinning intensity parameter A . Any neighbouring tree j being located closer to tree i than the critical distance cd_{ij} needs to be removed. Apart from the thinning intensity parameter A the index uses the height diameter ratio of tree i and the diameter of the

neighbouring tree j . The *A-thinning index* is sensitive to the h/d ratio of tree i : Trees with a larger h/d ratio are relatively more heavily released than those with a smaller h/d ratio. The values of A can range from 4 to 8 with decreasing thinning intensity. Johann (1982) recommended values of 4, 5 and 6 for even-aged pure Norway spruce forests which he considered to be synonymous with heavy, moderate and light release. A -values of 4 and 6 are frequently used values in thinning experiments (Hasenauer *et al.*, 1996; Pretzsch, 2002).

$$[1.14] \quad cd_{ij} = \frac{h_i}{A} \cdot \frac{DBH_j}{DBH_i}$$

Like competition indices (e.g. Biging and Dobbertin, 1992; Moravie *et al.*, 1999; Bauer *et al.*, 2004; Paulo *et al.*, 2002) the *A-thinning index* attempts to identify competitors of trees and to put thinning intensity on a quantitative basis. Hasenauer *et al.* (1996) have demonstrated how this index can be used as part of thinning event inventories to quantify tree competition before and after thinning. A can also be interpreted as a proportionality factor between the height of tree i and the critical distance. Pretzsch (2002) points out that this proportional relationship does not hold for young forest stands with small diameters and older stands with larger diameters. In the first case thinnings turn out to be too weak and the latter they become too heavy. His recommendation is to modify the formula cd_{ij} in the following way

$$[1.15] \quad cd_{ij} = \begin{cases} 2 \cdot \frac{DBH_i}{DBH_j}, \frac{h_i}{A} \leq 2m \\ 6 \cdot \frac{DBH_i}{DBH_j}, \frac{h_i}{A} \geq 6m \\ \frac{h_i}{A} \cdot \frac{DBH_j}{DBH_i}, \text{ otherwise} \end{cases}$$

The *A-thinning index* was originally developed for monospecies even-aged forests. For mixed species woodlands with different light requirements different A -values can be applied to each species or species group.

The *A-thinning index* can be applied differently by means of the re-arranged formula 1.16.

$$[1.16] \quad A_j = \frac{h_i}{dist_{ij}} \cdot \frac{DBH_j}{DBH_i}$$

For all $j = 1..n$ neighbours of tree i the individual A_j -value is calculated according to eq. 1.16 where $dist_{ij}$ is the observed distance between tree i and neighbour j . The larger is A_j the stronger the competition pressure of tree j on tree i . The n neighbours can then be sorted from high to low values according to their A_j -values. A decision can be made to remove only one, two or three competitors from the top of this list (Pretzsch, 2002). The *A-thinning index* is often used in individual tree growth simulators to quantify competition and to simulate selective thinnings (Hanewinkel and Pretzsch, 2000; Pretzsch *et al.*, 2002; Hasenauer, 2006). The index also has a high educational value when developing practical experience with selective thinnings.

1.2 Important forest stand or population variables

Forest stand variables are *summary statistics* that attempt to summarise tree data in a useful informative way. Summary statistics provide a numeric overview of tree and forest data and aim to describe them by numbers, functions or diagrams. They usually comprise means, standard deviation, minima, maxima, range, coefficient of variation and also frequency distributions (Porkress, 2004). Forest science and silviculture have developed a number of very specific characteristics. The most important ones are briefly discussed in this section. In mixed species forests it is very useful to describe each species separately.

The most traditional variable are *basal area (G)*, *trees or stems per hectare (SPH)* and the *mean squared diameter (dg)*.

English: trees/stems per hectare; *Welsh:* cyffion yr hectar; *German:* Stammzahl pro Hektar; *Russian:* число деревьев/стволов на 1 га; *Spanish:* pies por hectárea; *French:* nombre de tiges par hectare.

The basal area G of a forest stand is defined as the sum of the basal area of all individual trees related to one hectare (see eq. 1.17).

$$[1.17] \quad G = \frac{\sum_{i=1}^N g_i}{A}$$

where

- g_i basal area of tree i in m^2
- N number of trees in a forest stand
- A area of the sample plot(s) in ha

In long-term silvicultural research the *mean basal area over a period* (Assmann, 1970) is defined as the mean of the basal area between two thinnings weighted by the length of the increment period.

$$[1.17] \quad \bar{G} = \frac{\frac{G_{b1} + G_{e1}}{2} \cdot m_1 + \dots + \frac{G_{bn} + G_{en}}{2} \cdot m_n}{m_1 + \dots + m_n}$$

where

G_b basal area at the *beginning* of an increment period immediately *after* a thinning

G_e basal area at the *end* of an increment period immediately *before* a thinning

m number of years of an increment period, number of years between two thinnings

Stems per hectare (SPH) is simply

$$[1.18] \quad SPH = \frac{N}{A}$$

Care has to be taken when trees are measured in sample plots with variable plot size A . In that case each tree has its one area variable A_i .

Tree and basal area per hectare are often used to describe the density of a forest stand though *Trees per hectare* is not always an appropriate measure of site occupancy, since trees can be very different in size. *Basal area* is a useful measure of stocking level as it incorporates both size and number of trees, thus indicating how completely a piece of land is occupied by trees (Gadow and Bredenkamp, 1992). Stand density is a measure of the number of trees on a given piece of land whereas stocking refers to the degree of site occupancy.

English: stocking (degree, grade); *Welsh*: stocio; *German*: Bestockungsgrad; *Russian*: полнота древостоя; *Spanish*: densidad, existencias; *French*: degré de boisement.

Stocking degrees SD are variables derived from basal area and have the general formula

$$[1.19] \quad SD = \frac{G_{obs}}{G_{norm}}$$

where

G_{obs} observed forest stand basal area in m^2

G_{norm} “normal” forest stand basal area in m^2 derived either from yield tables or from an unmanaged control plot

While in the past normal basal area was derived from yield tables it is now increasingly derived from unmanaged control plots on the same site type and with the

same species composition. From such plots the norm is derived as the biologically maximum possible basal area. The corresponding stocking degree, which is often referred to as *natural stocking degree*, indicates the degree to which the maximum possible basal area has been reduced by management activities. This information is very relevant to silvicultural objectives. Forest stands not at the optimal stocking do not make the best use of the site and increment decreases. This can happen in both over-stocked ($SD > 1$) and under-stocked ($SD < 1$) forest stands. Stocking degrees are also strongly correlated with light levels available for natural regeneration and ground vegetation. The regulation of stand density is therefore an important function of woodland management (Gadow and Bredenkamp, 1992). Sterba (1975, 1981, 1987) developed methods to estimate the maximum possible basal area for a given site when this information is not available from long-term research plots.

There is an important relationship between stand density and average tree size on any given site and species. Populations of even-aged trees growing at high densities are subject to density-dependent mortality or self-thinning. For any given mean diameter there is a limit to the number of live trees per hectare that may coexist in even-aged stands. Ecologically this limiting relationship can be interpreted as the carrying capacity of any given site (Kimmins, 1997). The mean diameter in an unthinned plantation of a given initial density will increase until the limiting relationship is reached. Thereafter, further growth will be accompanied by mortality (Gadow and Bredenkamp, 1992). Among the first to describe this phenomenon was Reineke (1933). Yoda *et al.* (1963) developed $-3/2$ power law, which is based on the same idea (Pretzsch and Biber, 2005). Figure 1.9 illustrates the limiting relationship for two yield classes of Sitka spruce in Britain.

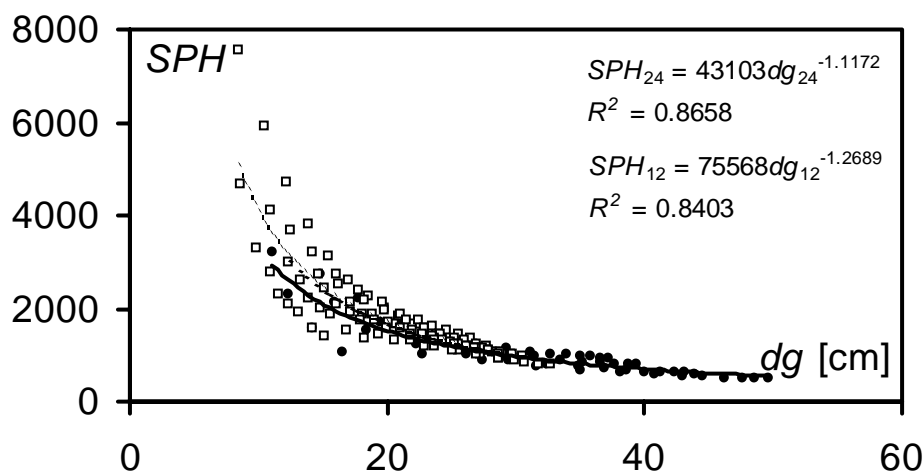


Figure 2.9. The limiting relationship for Sitka spruce (yield classes 12 and 24) in Britain. Rectangular, unfilled dots and dotted line – yield class 12, circular filled

dots and continuous line- yield class 24. Derived by the author from data of unthinned monitoring plots used to establish the British yield table system (Hamilton and Christie, 1973).

The relationship between stand density and average tree size is very important for woodland managers who wish to optimise thinning regimes. A large basal area is desirable for pulpwood and wood fuel stands while stands where sawtimber production is the prime objective large diameters are more important. Variations of this relationship have frequently been used in the past to produce silvicultural guide curves. A number of other density and stocking measures such as the stand density index (SDI) and the relative spacing have been derived from this relationship (Gadow and Breidenkamp, 1992; Laar and Akça, 1997).

English: mean squared diameter, quadratic mean diameter; *Welsh:* diamedr cymedrig cwadratig; *German:* Durchmesser des Grundflächenmittelstamms; *Russian:* средний диаметр, соответствующий площади сечения среднего дерева; *Spanish:* diámetro medio cuadrático; *French:* diamètre de l'arbre de surface terrière moyenne.

There is a number of different mean diameters in forest science. The most commonly used concept is that of the *mean squared diameter* or *quadratic mean diameter* d_g (see eq. 1.20).

$$[1.20] \quad d_g = \sqrt{\frac{\sum_{i=1}^N d_i^2}{N}} = 200 \cdot \sqrt{\frac{\sum_{i=1}^N g_i}{\pi \cdot N}}$$

where

d_i breast height diameter [cm] of tree i

g_i basal area [m²] of tree i

The mean squared diameter represents the tree with the mean basal area. The mean squared diameter forms an arithmetic system with trees per hectare and basal area, so that any of these three variables can be calculated when two of them are known (see eq. 1.21).

$$[1.21] \quad SPH = \frac{40000}{\pi} \cdot \frac{G}{dg^2}; \quad G = SPH \cdot dg^2 \cdot \frac{\pi}{40000}; \quad d_g = \sqrt{\frac{40000}{\pi} \cdot \frac{G}{SPH}}$$

In the British yield tables the mean arithmetic diameter is used, which is normally restricted to investigations of seedling and sapling populations only. Other population diameters include the central basal area diameter and the top height diameter which will be discussed later.

$$[1.22] \quad V = \frac{\sum_{i=1}^N v_i}{A}$$

Volume per hectare is derived in a similar way to trees and basal area per hectare (eq. 1.22). Volume is often used as a density measure in older forests and plays an important role in silvicultural management guidelines. Forest stand volume can also be estimated by multiplying the volume corresponding with the quadratic mean diameter with SPH (eq. 1.23).

$$[1.23] \quad \hat{V} = v_g \cdot SPH$$

where

v_g volume of the quadratic mean diameter tree

\hat{V} estimated forest stand volume

Eq. 1.23 again emphasises the significance of the quadratic mean diameter d_g .

English: diameter distribution; *Welsh:* dosbarthiad diamedrau; *German:* Durchmesservertelung; *Russian:* распределение деревьев по ступеням толщины; *Spanish:* distribución diamétrica; *French:* répartition des diamètres.

Naturally diameter distributions offer more information than population diameters. The shape of diameter distributions is related to the structure of forest stands. Empirical frequency distributions are commonly computed as diagrams with equal class intervals in which the vertical scale is frequency. With diameter distributions equidistant class widths are common. The choice of the ideal class width is a serious statistical problem and various formulae have been suggested to optimize class width. In forest science and practice 1, 2, 4, 5 and 10cm classes are common. Most frequently, however, 1 and 4cm classes are used.

1cm-diameter classes		4cm-diameter classes	
...		...	
Class 8: $7.5cm \leq DBH_i < 8.5cm$	[7.5, 8.5)	Class 12: $10.0cm \leq DBH_i < 14.0cm$	[10.0, 14.0)
Class 9: $8.5cm \leq DBH_i < 9.5cm$	[8.5, 9.5)	Class 16: $14.0cm \leq DBH_i < 18.0cm$	[14.0, 18.0)
...		...	

The number of each diameter class corresponds with its mid-point diameter. The choice of class width should be made dependent on the objectives of the analysis. Figure 1.10 illustrates the case of two different diameter classes applied to a mixed species woodland at Clocaenog forest.

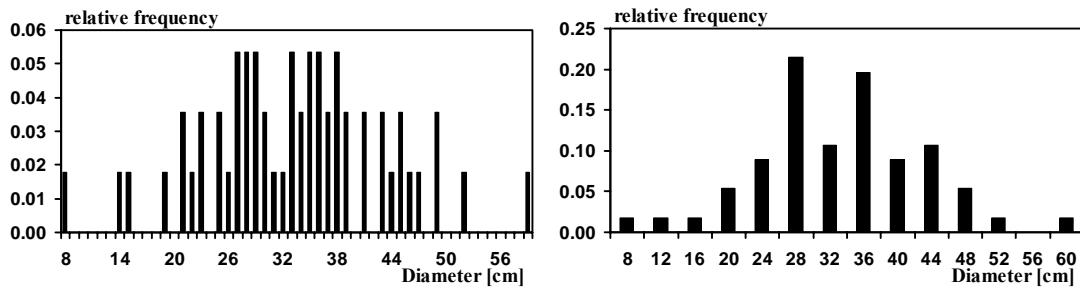


Figure 1.10. Diameter distributions from a mixed Sitka spruce – lodgepole pine forest stand at Clocaenog forest (North Wales). Left: 1cm-classes, right: 4cm-classes.

The comparison intuitively shows that 4cm-classes are more appropriate to get a better impression of the diameter structure.

An efficient way to compute equidistant frequency distributions is to use eq. 1.24. Each tree diameter is divided by the class width and the result is converted to an integer number. This is the number of the diameter class the tree belongs to. Finally the number of trees in each class can be counted.

$$[1.24] \quad class = \text{int} \left[\frac{d}{class\ width} \right]$$

Another way of further summarizing diameter distributions is to fit models to their data. The most common models include *Weibull*, *beta* and *gamma* distributions (Laar and Akça, 1997; Philip, 1994). The parameters of such distributions functions can be used as structural indices.

English: (stand) height curve, height diameter curve; *Welsh*: cromlin uchder/diamedr; *German*: (Bestandes)höhenkurve; *Russian*: кривая высот (деревьев); *Spanish*: curva altura-diámetro; *French*: courbe de répartition des hauteurs.

There is a non-linear statistical relationship between diameters and heights of a tree population, which has underlying physical and physiological causes. The nature of the relationship is evident when depicting measured tree heights over the corresponding diameters in a diagram. As diameter increases tree height increases at a lower rate until height curve finally levels off (in mathematical terms the height curve is approaching an asymptote). As it is more complicated to take height measurements than diameter measurements, it is quite common to measure heights on a sample basis only. A general recommendation is to sample between 30-40 tree heights across the range of diameters per species per population (e.g. a forest stand). Especially for the assessment of interspecific competition, volume and for research purposes the height/diameter relationship of a tree population should be analysed separately for

each species. A number of models have been developed for the quantitative description of the height/diameter relationships. The most important ones are listed in Table 1.5.

Table 1.5. Selected height curve functions. a_0 and a_1 are regression coefficients. d – diameter at breast height [cm], h – (total) tree height [m], \ln – natural logarithm, e – base of the natural logarithm.

Eq. #	Name	Main version	Linear version
[1.25]	Logarithmic	$h = a_0 + a_1 \cdot \ln d$	$h = a_0 + a_1 \cdot \ln d$
[1.26]	Petterson (1955)	$h = 1.3 + \left(\frac{d}{a_0 + a_1 \cdot d} \right)^3$	$(h - 1.3)^{-\frac{1}{3}} = a_0 + a_1 \cdot \frac{1}{d}$
[1.27]	Michailoff (1943)	$h = 1.3 + a_0 \cdot e^{-\frac{a_1}{d}}$	$\ln(h - 1.3) = \ln a_0 + a_1 \cdot \frac{1}{d}$
[1.28]	Oliveira cited from Gadow & Bredenkamp(1992)	$h = e^{a_0 + a_1 \cdot \frac{1}{d}}$	$\ln h = a_0 + a_1 \cdot \frac{1}{d}$

By transforming height curve equations to a linear form the regression coefficients can be calculated through linear regression. Height curves are useful tools when assessing interspecific competition and the vertical structure of a forest (see Figure 1.12). Height curve functions help to estimate total tree heights for trees where only diameter information is available. Height curves are also the basis for the estimation of forest population heights, *mean stand heights*, *top heights* and volume. Figure 1.11 illustrates the height diameter functions of Table 1.5.

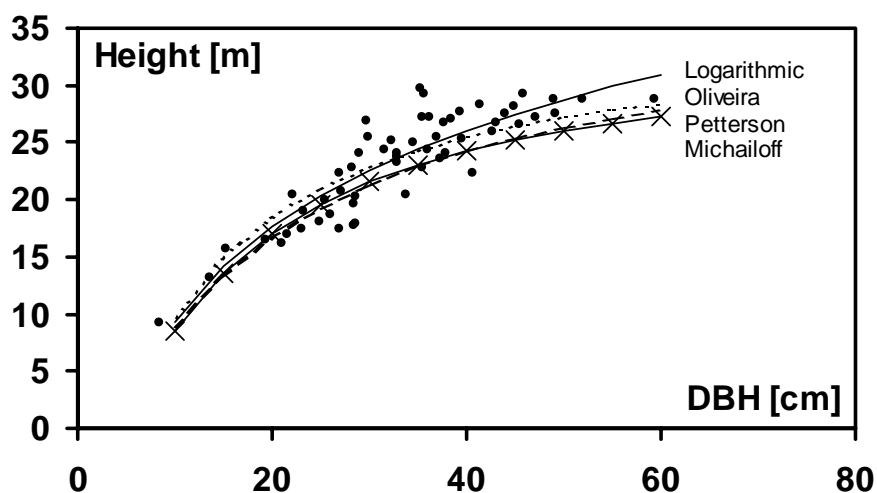


Figure 1.11. Height diameter curves of Table 1.4 for a mixed Sitka spruce – lodgepole pine forest stand at Clocaenog forest (North Wales).

The logarithmic function is one of simplest height functions but quite inflexible and therefore especially unsuitable in diverse forests. The Oliveira function is similar but more flexible than the logarithmic function. Petterson and Michailoff functions have turning points which allow a better adaptation to diverse, irregular forests.

Figure 1.12 illustrates the use of stand height curves to assess interspecific competition. The light demanding lodgepole pine is overtopped by Sitka spruce. Lodgepole pine cannot survive if intimately mixed with Sitka spruce. Only a spatial segregation of the two species provides a chance for lodgepole pine. In fact in the forest under study lodgepole pine only occurs on dry outcrops where the species had a long-term advantage over Sitka spruce. The graphs also show that the forest potentially has two canopy layers.

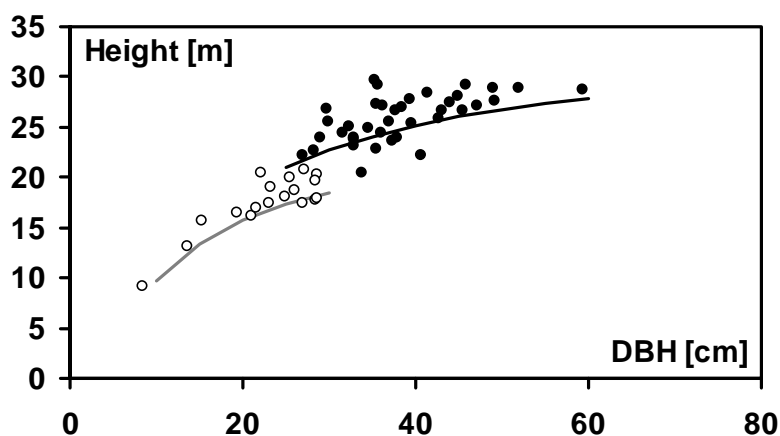


Figure 1.12. Height diameter curves of Sitka spruce (black) and lodgepole pine (white, grey) for a mixed Sitka spruce – lodgepole pine forest stand at Clocaenog forest (North Wales).

Population or forest stand height is a useful response variable in silvicultural trials and tree breeding experiments. Unlike tree diameters it reacts more to site condition than to management induced competition.

Like the arithmetic mean diameter the arithmetic mean height is heavily affected by thinning operations. It is therefore only applied in biological investigations involving very young plantations (fertilizer, provenance experiments).

Usually population heights are derived from height curves by inputting population diameters into the height equation. The mean height of a tree population is often used to estimate the volume of the quadratic mean diameter tree and then the volume per hectare. Mean heights and age were once used as input variables for yield tables but in most modern yield tables mean heights have been replaced by top heights which are less susceptible to thinning effects.

English: top height, dominant height; *Welsh:* uchder brig; *German:* Oberhöhe, Spitzenhöhe; *Russian:* доминирующая высота; *Spanish:* altura dominante; *French:* hauteur maximale, hauteur dominante.

Top and mean heights are derived from the *top height diameter* or d_{100} (defined as the mean squared diameter of the 100 largest trees/ha) and the mean squared diameter d_g , respectively. Figure 1.13 shows the estimation procedure graphically.

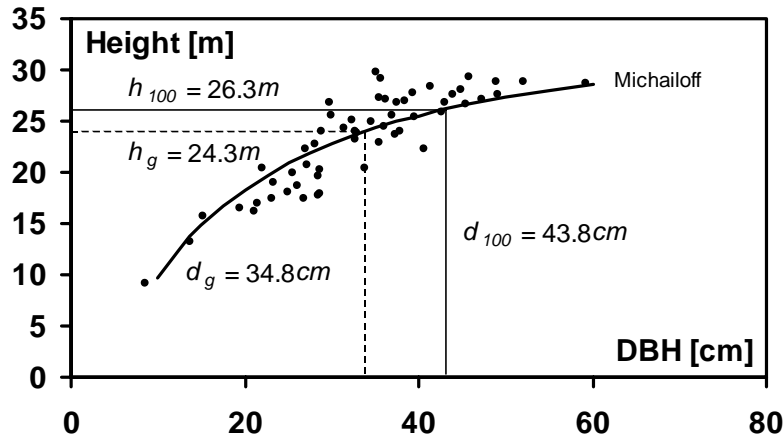


Figure 1.13. Illustration of the estimation of mean and top height from the mean squared diameter, the top height diameter and the height curve equation. If sufficient species specific measurements are available population diameters and heights can also be estimated species specifically.

The ratios of h_g/d_g and h_{100}/d_{100} are often used as population values of the h/d ratio. In conifers the latter is especially important since it represents the h/d ratio of the most dominant trees, which are the trees most exposed to wind in a forest stand. In Britain, Forest Research has adopted a procedure of estimating top heights, which is different from the international standard. Other authors have pointed out, that top or dominant height diameters can be more precisely defined statistically as when calculated as the mean squared diameter of a relative number of trees. Weise for example defined the top height diameter as the mean squared diameter of the 20% largest trees, which corresponds approximately with $d_{90\%}$, i.e. the 90% percentile of the cumulative diameter distribution. There is some evidence that the mean squared diameter of the 10% biggest trees, corresponding with the 95% percentile of the cumulative diameter distribution, $d_{95\%}$, is an even better definition of the top height diameter (Wenk *et al.*, 1990). In some text books top heights are referred to as *dominant heights*.

Top height is an important indicator of site quality. The *site index*, for example, is defined as the top height attained at some specified reference age, e.g. 30 years. For this purpose it is necessary to establish the top height development over age or alternatively to sample top height exactly at the time of the reference age. Figure 1.14 shows the top height assessment for three Welsh long-term sample plots. According to this Glasfynydd forest exhibits the best top height development and

highest site index signifying the best site conditions for Sitka spruce of all three locations.

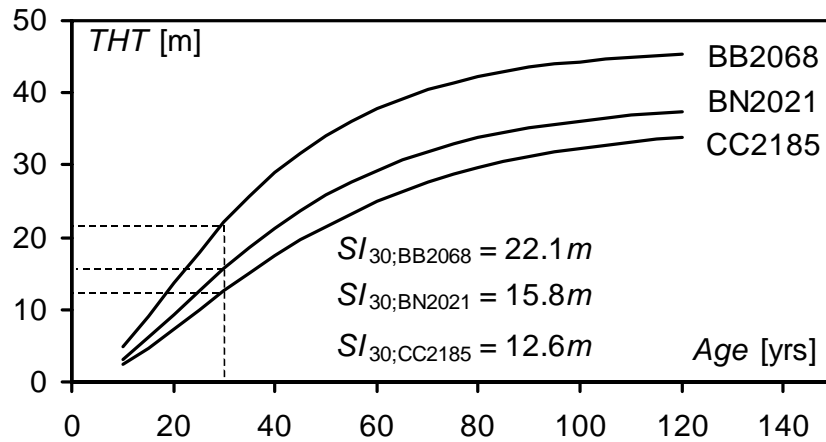


Figure 1.14. The site index at the age of 30 years (SI_{30}) of three Sitka spruce forests in Wales. THT – top height, BB2068 - Glasfynydd Forest, BN2021 - Coed y Brenin, CC2185 – Clocaenog forest.

Individual tree structural indices, as introduced in the previous section, can also be aggregated to give summary characteristics. They can be expressed as arithmetic mean values and empirical frequency distributions. Spatial summary characteristics using data from finite sample plots often require edge corrections to provide unbiased estimation (Pommerening and Stoyan, 2006). The mean uniform angle index is therefore calculated as

$$[1.29] \quad \bar{W} = \frac{1}{N} \sum_{i=1}^N W_i$$

where

N number of trees in the forest

W_i uniform angle index of tree i

and the mean of the mean directional index, the mean mingling index, the mean differentiation and the mean dominance index are calculated in a similar way. The line above the index symbol symbolises the fact that it is an arithmetic mean.

More informative are the corresponding index distributions with individual trees put into classes. For the uniform angle, mean directional, mingling and dominance index there are always as many classes as there are discrete individual tree index values, i.e. $n + 1$ with n being the number of neighbours. For example as shown in Figure 1.7 M_i , can take 5 different values with $n = 4$: 0.00, 0.25, 0.50, 0.75 and 1.00. In order to construct a distribution all trees are sorted into their respective class

according to their M_i values. Figure 1.15 illustrates the mingling distribution for the species mixed woodland Pen yr Allt Ganol in Gwydyr forest (North Wales).

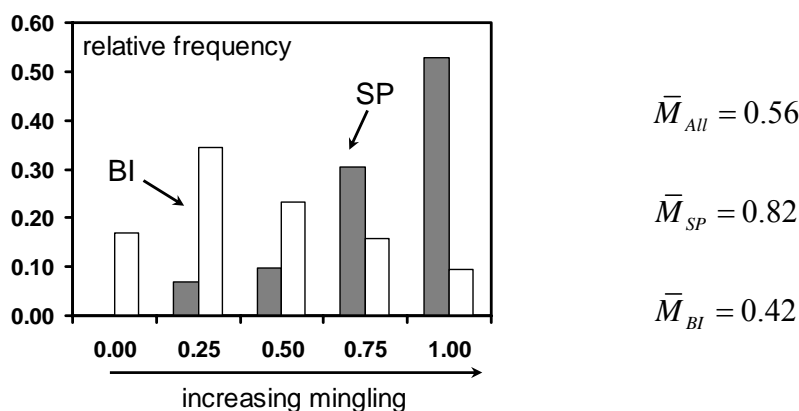


Figure 1.15. Mingling distribution (left) of Scots pine (SP) as the main species of the overstorey and of birch (BI) as the main species of the understorey. The corresponding mean arithmetic values on the right.

The mingling distributions clearly show that Scots pine and birch have an opposite mingling behaviour. While Scots pine trees are mostly surrounded by trees of another species birch trees tend to be arranged in clusters. Similar graphs can be produced for the other neighbourhood indices. Differentiation distributions are constructed following the following principle

- Class 1: Small differentiation:** comprises the classes $0.0 \leq TD_i, TH_i < 0.3$. The tree with the smallest DBH/total height has 70 % or more of the neighbouring tree's size.
- Class 2: Medium differentiation:** comprises the classes $0.3 \leq TD_i, TH_i < 0.5$. The tree with the smallest DBH/total height has 50 - 70 % of the neighbouring tree's size.
- Class 3: Large differentiation:** comprises the classes $0.5 \leq TD_i, TH_i < 0.7$. The tree with the smallest DBH/total height has 30 - 50 % of the neighbouring tree's size.
- Class 4: Very large differentiation:** comprises the classes $0.7 \leq TD_i, TH_i < 1.0$. The tree with the smallest DBH/total height has less than 30 % of the neighbouring tree's size.

Figure 1.16 illustrates the diameter differentiation distributions for Pen yr Allt Ganol. As Scots pine trees with large diameters are often surrounded by regeneration trees with small diameters the diameter differentiation of Scots pine tends to be very large. Birch trees on the other hand are often surrounded by other birch trees or other regeneration trees with small diameters so that the diameter differentiation of birch tends to be small.

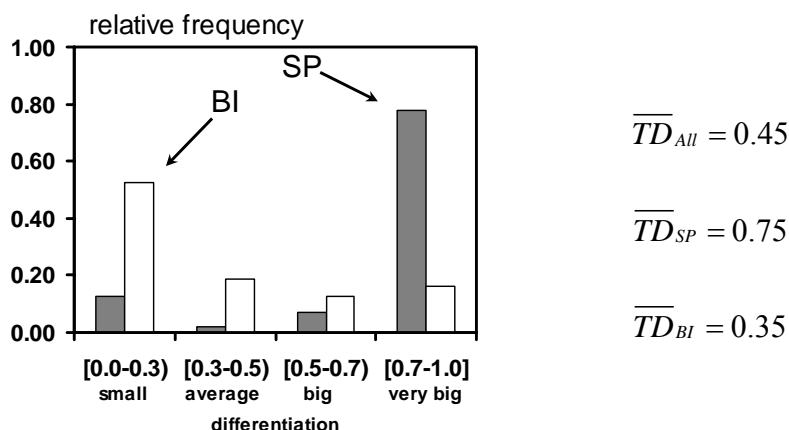


Figure 1.16. Diameter differentiation distribution (left) of Scots pine (SP) as the main species of the overstorey and of birch (BI) as the main species of the understorey. The corresponding mean arithmetic values on the right.

Gadow and Hui (2002) have suggested using the classes of the dominance distribution as spatial variants of relative crown classes. Relative in this context means that trees are not classified according to overall canopy layers as in Kraft's crown classes but rather classified individually in the context of their nearest neighbours.

Table 1.5. Interpretation of the dominance index in the case of $n = 4$ neighbours.

U_i	Description	Relative crown class
0.00	4 of 4 neighbours smaller than reference tree	very suppressed
0.25	3 of 4 neighbours smaller than reference tree	moderately suppressed
0.50	2 of 4 neighbours smaller than reference tree	co-dominant
0.75	1 of 4 neighbours smaller than reference tree	dominant
1.00	0 of 4 neighbours smaller than reference tree	strongly dominant

Figure 1.17 illustrates the significance of "relative" crown classes. The majority of Scots pine trees are dominant in the context of their neighbours since they are always surrounded by much smaller regeneration trees. Birch trees occur in clusters, some of their neighbours are overstorey Scots pine trees, some are smaller or larger birch trees or other broadleaves. As a result birch trees are almost evenly distributed over the five dominance classes. Kraft's crown classes, however, would classify almost all birch trees as 5a or overtopped trees.

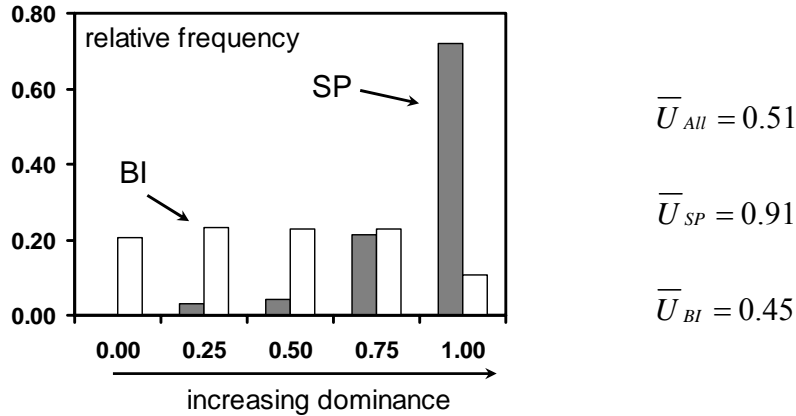


Figure 1.17. Dominance distribution (left) of Scots pine (SP) as the main species of the overstorey and of birch (BI) as the main species of the understorey. The corresponding mean arithmetic values on the right.

The *aggregation index* of Clark and Evans (1954) is a summary characteristic describing the diversity of tree position. The index does not have an individual tree equivalent. The aggregation index is defined as

$$[1.30] \quad R = \frac{\bar{r}_{observed}}{E(r)} \quad \text{where} \quad E(r) = \frac{1}{2 \cdot \sqrt{\frac{N}{A}}}; \quad R \in [0, 2.1491]$$

$\bar{r}_{observed}$ stands for the mean of the distances from the trees to their nearest neighbours in a given forest stand, while $E(r)$ is the mean nearest neighbour distance in a stand with completely random tree locations (“Poisson forest”). Usually, the interpretation of R -values is as follows:

- $R > 1$ if the pattern has a tendency to regularity,
- $R = 1$ if it is completely random (Poisson process),
- $R < 1$ if there is clustering in the pattern.

A further index accounting for spatial tree species diversity is the *coefficient of segregation*, S , of Pielou (1977). It describes the degree of mixing of trees of two species A and B in a forest, and like the aggregation index R , it is based on the nearest neighbour tree distances. In Pielou’s notation, S is given as follows:

Let N be the number of all pairs of trees (reference tree – nearest neighbour tree), let m and n be the numbers of trees of species A and B , respectively, and let r and s be the number of times trees of species A and B are found as the nearest neighbours of a reference tree. These numbers can be set out in a simple form in a 2×2 table as follows.

		species of the nearest neighbour		
		A	B	total
species of reference tree	A	a	b	m
	B	c	d	n
	total	r	s	N

The coefficient of segregation is defined by

$$[1.31] \quad S = 1 - \frac{N \cdot (b + c)}{m \cdot s + n \cdot r}; \quad S \in [-1, 1]$$

If the nearest neighbours are always of the same species as the reference trees, then $S = 1$. If all neighbours are of different species $S = -1$. In the case of complete randomness of species distribution, one can expect values around 0.

The Shannon index³ (Shannon and Weaver, 1949) is an example of a non-spatially explicit algorithm describing species or size diversity. It is defined by

$$[1.32] \quad H' = - \sum_{j=1}^n p_j \cdot \ln(p_j)$$

where

p_j probability of a randomly selected tree belonging to tree species j or to diameter class j

n number of tree species or diameter classes in the forest.

p_j can be calculated as relative frequency of tree species either in terms of number of trees or basal area. Using basal area gives an additional aspect of density. The Shannon index takes the relative abundance of different species/size classes into account rather than simply expressing species/size richness. The rationale of this index is based on information theory. It measures the uncertainty of the next letter in a coded message or the next species to be found in a community. A monospecies forest would have no uncertainty and $H' = 0$. The Shannon index is affected by both the number of species/diameter classes and their equitability or evenness. The index is particularly sensitive to the abundances of rare species in the community.

In order to be able to compare the diversity of different tree populations the Shannon index is often converted to the evenness index (eq. 1.33).

³ In the original publication the base 2 logarithm (\log_2) was used. The base e logarithm is, however, preferred in many studies.

$$[1.33] \quad E_{H'} = \frac{H'}{H'_{\max}} = \frac{H'}{\ln n}$$

By definition, the evenness index is constrained between 0 and 1 while the Shannon index can take a wide range of different values (Krebs, 1999).

Based on the Shannon index Pretzsch (1998) developed the species profile index. This index accounts for relative species abundances in three different height zones ranging from 0-50%, 50-80% and 80-100% of maximum total tree height equating with Assmann's definition of overstorey, mid storey and understorey (Assmann, 1970, Table 1.4).

$$[1.34] \quad A = - \sum_{i=1}^n \sum_{j=1}^z p_{ij} \cdot \ln(p_{ij})$$

where

p_{ij} proportion of species i in height zone j

n number of tree species in the forest.

z number of height zones

The species profile index summarises and quantifies species diversity and the vertical distribution of species in a forest. While the index is lowest in one-storeyed pure forests, it rises for pure forests with two or more storeys. Peak values are reached in mixed woodlands with heterogenous structures (Hanewinkel and Pretzsch, 2000). Evenness of the species profile index is calculated as

$$[1.35] \quad E_A = \frac{A}{A_{\max}} = \frac{A}{\ln(n \cdot z)}$$

The index by Simpson (1949) gives the probability that any two individuals drawn at random from an infinitely large population belong to different species. Simpson suggested that this probability was inversely related to diversity. The Simpson index is considered a dominance index because it is weighted towards the abundance of the most common species. The index can also be applied to tree size diversity (diameters, height). In the literature the index has two base types:

$$[1.36] \quad D = \sum_{m=1}^n \frac{N_m^2}{N^2} = \sum_{m=1}^n p_m^2 \quad \text{or} \quad D = \sum_{m=1}^n \frac{N_m(N_m - 1)}{N(N - 1)}$$

where

D

Simpson-Index base

$$N_m \quad \text{number of individuals of species } m$$

$$N = \sum_{m=1}^n N_m$$

The final Simpson indices can be calculated as either complementary or reciprocal form (eq. 1.37).

$$[1.37] \quad 1 - D \text{ or } \frac{1}{D}$$

The original formula developed by Simpson is the left hand version of eq. 1.36 in conjunction with the left hand version of eq. 1.37. Krebs (1999) briefly discusses the advantages and disadvantages of the different versions. The corresponding evenness index to Simpson's index is

$$[1.38] \quad E_D = \frac{D}{n}.$$

where

$$D = 1 - \sum_{m=1}^n p_m^2$$

Other structural indices are described in detail in Krebs (1999), Staudhammer and LeMay (2001), Neumann and Starlinger (2001), Pommerening (2002, 2006), Pommerening and Stoyan (2006) and Stoyan and Penttinen (2000).

English: growing space; *Welsh:* gofod tyfu; *German:* Wuchsraum, Standraum; *Russian:* площадь питания дерева, пространство роста; *Spanish:* espacio de crecimiento; *French:* ???.

Since competition between trees is an important aspect of silviculture much attention has been paid to the quantification of available *growing space*. Methods applied are very diverse and range from crown projection areas and the *growing space index*⁴ (Assmann, 1970) to *Voronoi* and *Dirichlet tessellations* (Stoyan and Stoyan, 1994). A good overview of methods is given in Pretzsch (2002) and Wenk *et al.* (1990).

English: leaf area index; *Welsh:* indecs arwynebedd dail; *German:* Blattflächenindex; *Russian:* индекс площади листьев, индекс листовой поверхности; *Spanish:* índice de área foliar; *French:* ???.

⁴ The growing space index is identical with the linear crown index or *k/d* ratio (eq. 2.7 in Table 2.2) and defined as crown diameter divided by tree diameter. In his investigation on the growth of wild cherry trees Spiecker (1994) detected a strong linear relationship between the growing space index and the diameter growth of individual trees. The index is comparatively easy to sample and offers a fairly robust and feasible measure of the growth potential of individual trees.

An alternative to the quantification of growing space is the *leaf area index* (Waring, 1983; O'Hara, 1996). It is defined as the projected leaf surface area per unit of ground surface area (Kimmins, 1997; Laar and Akça, 1997). The *leaf area index* $LAI = 0$, if there are no leaves at all. If the leaf area corresponds to the soil surface area, $LAI = 1$. If the leaf area is twice the soil surface, $LAI = 2$. The maximum value of $LAI = 16$ is reached in the ever-green forests of west coast of the USA. Leaf area indices of 2.14 and 2.59 have been recorded by Mencuccini and Grace (1994) in Thetford and Aberfoyle forests for Scots pine. The leaf area index is closely related to photosynthesis and is also to the water balance of a site. The drier the site, the lower the leaf area. Grier and Waring (1974) found that leaf area is closely related to the basal area of sapwood. Each tree species has a specific leaf area-sapwood basal area relationship. Once the relationship for a species has been established foliage surface area per tree can easily be estimated simply by boring the tree to obtain an increment core. Similar conclusions have been made for example by Mencuccini and Grace (1994) in the United Kingdom. Leaf area also exhibits some interesting relationships with stemwood biomass on both an individual tree and a stand basis (Kimmins, 1997). O'Hara *et al.* (1999) and O'Hara *et al.* (2001) have suggested leaf area allocation as a woodland management guide. Dufrêne and Bréda (1995) and Wijk and Williams (2005) give good overviews of direct and indirect methods of measuring the leaf area index.

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