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A yield prediction model for pure and mixed stands

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Abstract

A model for growth and yield in pure and mixed stands is described, in which stand development is derived from a description of increment in relation to age and thinning management. The initial condition for the model is described by stand volume, stocking density, and stand height at a certain stand age. Height development is related to volume increment through an allometric relationship; thinning volume depends on the degree of thinning, as well as on the production level as expressed in the relationship between increment and stand age. The model was calibrated on permanent field plot data, and tested against independent long term growth and yield records. The presented version of the model has been used to construct yield tables that are currently being used in eastern Germany. Possibilities for model extensions are discussed, together with the possibilities for model improvement.

Introduction

Over the past decades, a general growth and yield model has been developed at the Institute for Wood Growth and Forest Informatics, aimed at the simulation and prediction of forest growth and yield in pure and mixed stands. The basic growth algorithm was first described by Wenk (1972) and subsequently modified and expanded. The basic analysis underlying the growth and yield model was described in detail by Wenk et al. (1990).

Model description

Volume increment

The central element of the model is the algorithm for describing stem volume increment in relation to stand age as illustrated in Fig. 1. Stem volume at any age can be taken as the initial condition at the beginning of the simulation. In case of thinning immediately at the beginning of the simulation, the

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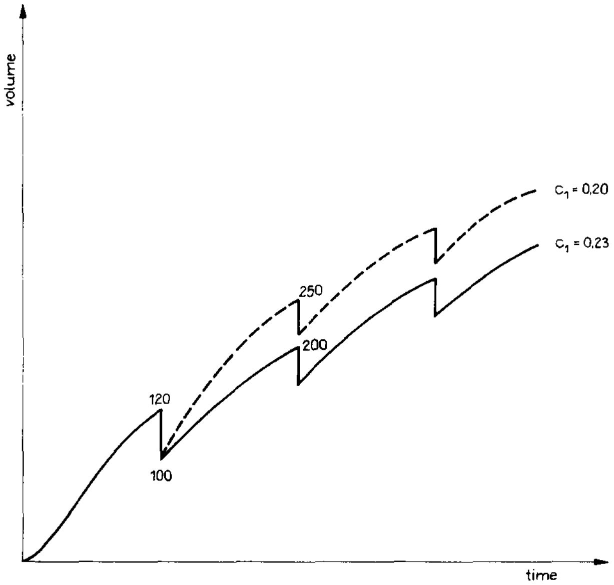

Fig. 1. Schematic representation of stand volume development in relation to stand age, for two values of the growth parameter c_1.

amount of stem volume removed by thinning is accounted for, and the remaining stock is taken as the new initial condition. The annual increment is derived from actual stand volume through multiplication by a growth multiplier, which is derived from observed stem volume at successive measurements

$$M_{t+\Delta t} = V_{b,t+\Delta t} / V_{a,t} \tag{1}$$

where $M_{t+\Delta t}$ is the growth multiplier at time $t+\Delta t$ (t in years), $V_{b,t+\Delta t}$ is stand volume before thinning at $t+\Delta t$ (m^3 ha^{-1}) and $V_{a,t}$ is stand volume after thinning at time t (m^3 ha^{-1}).

Using Eq. (1), total stem volume at $t+\Delta t$ is calculated as (in m^3 ha^{-1} over the period to which $M_{t+\Delta t}$ applies)

$$V_{t+\Delta t} = M_{t+\Delta t} \dot{V}_t \tag{2}$$

This algorithm is repeated in the model until the end of the simulation. The multiplier $M_{t+\Delta t}$ can now be expressed as an apparent relative growth rate for which a general expression can be derived. Suppose

$$M_{t+\Delta t} = 1/(1 - P_{t+\Delta t}) \tag{3}$$

with

$$P_{t+\Delta t} = (V_{b,t+\Delta t} - V_{a,t}) / V_{b,t+\Delta t} \tag{4}$$

The current periodic increment ($I_{V,t+\Delta t}$) is the numerator which is divided by the stand volume at the end of the period. If Δt goes to zero, then $P_{t+\Delta t}$ approaches the relative volume growth rate. Thus, the growth multiplier $M_{t+\Delta t}$ can be derived from a description of relative growth rate over time (see Wenk, 1973, or Davids, 1987, for further detail). For the function P_t, a range of expressions can be chosen, e.g. the expression derived by Wenk (1969)

$$P_{V,t*}=\exp\{-c_1 t^*[1-\exp(-c_2 t^*(1-\exp(-0.4t^*)))]\} \quad (5)$$

with $P_{V,t*}$ indicating that the values calculated with Eq. (5) are estimates for relative volume increment of trees and stands. The parameters $c1$, $c2$ and $c3$ represent regression coefficients to be derived from permanent plot data. P_V is defined by Eq. (4), with Δt in Eq. (4) amounting to a fixed period between field measurements, e.g. 10 years. As an example, in an 80-year-old stand ($t+\Delta t=80$), with a stem volume ($V_{b,t+\Delta t}$) of 400 m^3, and a value for $P_{V,t}$ of 0.2, the periodic (10 year) increment just before age 80 would be 80 m^3 ($I_V(70\text{–}80)=400\times 0.2$ m^3 ha^{-1}). For this reason, age t in Eq. (5) has been transformed using $t^*=(t-10)/10$, with t in years.For $t\to\infty$ Eq. (5) converges to

$$P_{V,t*}=\exp(-c_1 t^*) \quad (6)$$

and for $\Delta t\to 0$, $P_{V,t*}$ becomes the relative growth rate. In that case, Eq. (6) can be integrated to give the Gompertz equation (Wenk, 1979). The parameter c_1 defines the shape of the growth curve, and depends on tree species, site quality and silvicultural treatment (see Fig. 2). For a given initial condition, c_1 determines the slope of the curve of stand volume over age (Fig. 1). Under constant conditions, the growth parameter c_1 is independent of age.

The parameter c_2 in Eq. (5) accounts for variability of juvenile growth, and usually loses its influence beyond age 40–50. Beyond the intersection of the curve defined by Eq. (5), and its collapsed form as given by Eq. (6), it is assumed that only parameter c_1 determines the shape of the curve (Fig. 3).

Height development

As in the case of stem volume, height development is calculated from an initial condition using a multiplier that depends on stand age

$$H_{t+\Delta t}=M_{H,t+\Delta t}H_t \quad (7)$$

where $H_{t+\Delta t}$ is stand height at time $t+\Delta t$ (m, t in years), $M_{H,t+\Delta t}$ is the height multiplier at time $t+\Delta t$, H_t is stand height at time t (m).

The multipliers for volume increment and height growth are related through an allometric equation (Wenk, 1978)

$$(1-P_V)=(1-P_H)^m \quad (8)$$

with P_H defined analogous to Eq. (3), using $M_{H,t}=1/(1-P_{H,t})$. Using the P_V

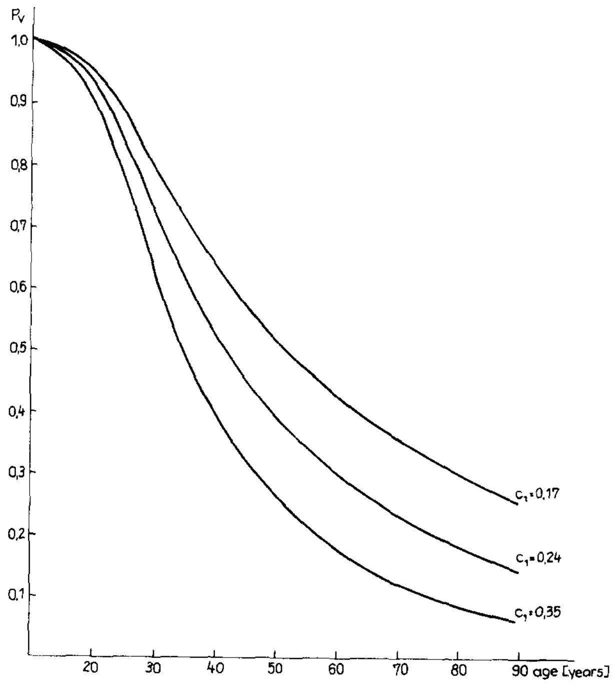

Fig. 2. The effect of growth parameter c_1 on P_V, as determined by Eqs. (5) and (6).

values from Eq. (5), height development can now be calculated provided the allometric coefficient m is known, and given an initial value for H_t. Thus, height growth is in fact completely controlled by volume growth.

Thinning

For the calculation of the amount of stem volume removed by thinning ($V_{th,t}=V_{b,t}-V_{a,t}$, in $m^3\ ha^{-1}$), thinning intensity is linked to the relative volume increment P_V from Eq. (5) through the following differential equation

$$d(P_{V,t}-P_{th,t})/dt=-k_V c_1 (P_{V,t}-P_{th,t}) \quad (9)$$

where $P_{V,t}$ is relative volume increment at time t, $P_{th,t}$ is thinning intensity at time t, expressed as the ratio between stem volume before and after thinning ($V_{a,t}/V_{b,t}$), t is stand age in years, k_V is thinning factor defining thinning intensity, c_1 is the species-dependent growth parameter, as in Eqs. (5) and (6).

Application of Eq. (9) assumes that stand volume does not decrease with increasing stand age. The integrated form of Eq. (9) is simply given by

$$P_{th,t*}=P_{V,t*}-\exp(-c_1 t^*) \quad (10)$$

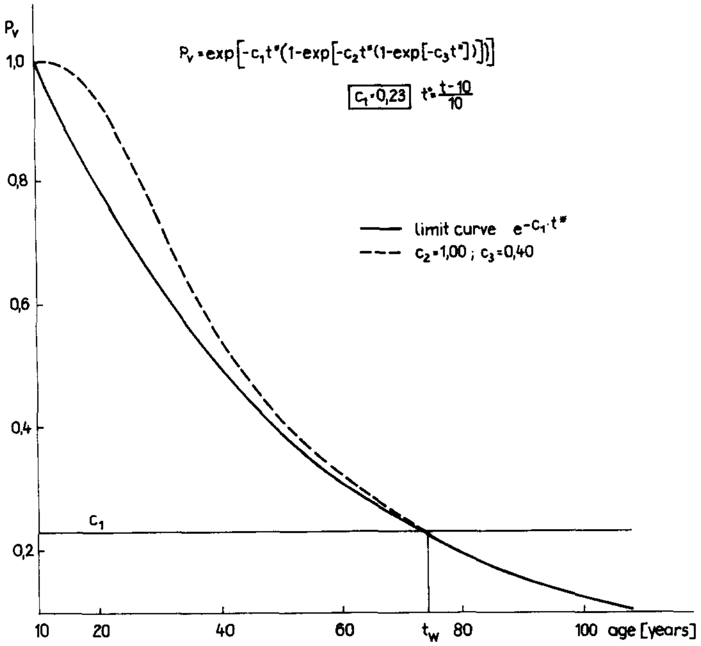

Fig. 3. Convergence of the relative increment (Eq. (5)) to the curve described by $P_{V,t*} = \exp(-c_1 t^*)$ (Eq. (6)).

where $P_{V,t*}$ and t^* are as in Eq. (6). Thus, for $k_V = 1$, thinning intensity becomes zero. For $k_V \rightarrow \infty$, $P_{th,t*}$ becomes equal to $P_{V,t*}$, and effectively all increment is harvested. This thinning model was extended by Zimmermann (1974) to allow for different thinning intensity during stand development, especially to account for variable thinning in young stands, using the expression

$$P_{th,t*} = P_{V,t*} - \exp[-c_1 t^* k_V (1 + a\exp(-bt^*))] \qquad (11)$$

where the parameters a and b define deviations of thinning intensity with age, relative to $P_{th,t*}$ as given by Eq. (10). Figure 4 represents Eq. (11) for a range of parameter values.

Using Eqs. (1–6) and (9–11), the total stem volume production, thinning yields, and current increments can now be calculated from an given initial condition onwards.

Stocking density

Stocking density changes concurrently with thinning. Here, it is described in much the same way as thinning, starting from an initial stand density in trees per hectare. To calculate the effect of thinning on stocking density, an

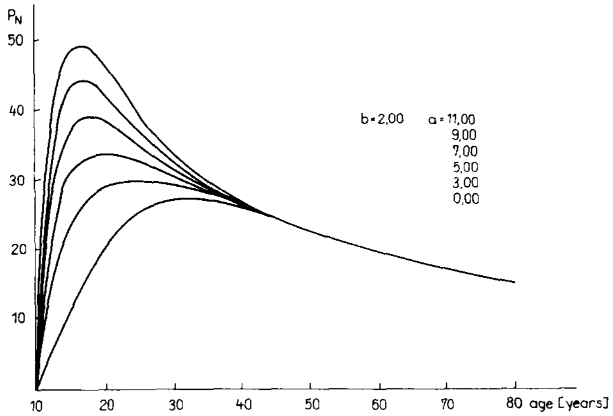

Fig. 4. Thinning intensity (expressed as a percentage) as determined by a constant parameter for k_V and b, and different values for parameter a.

additional thinning intensity expressed as the ratio between stocking density before and after thinning is applied

$$P_{th-N,t}=N_{th,t}/N_{b,t} \tag{12}$$

where $P_{th-N,t}$ is thinning intensity, expressed in terms of numbers of trees, at time t, $N_{th,t}$ is the number of trees removed at time t (ha^{-1}), $N_{b,t}$ is the number of trees before thinning at time t (ha^{-1}).

According to Zimmermann (1974), the thinning intensity expressed in number of trees can be related to thinning intensity expressed as stem volume from the ratio between the volume of the average tree of the thinning (v_{th}), and the volume of the average tree before thinning (v_b)

$$P_{th}/P_{th-N}=v_{th}/v_b=k_N \tag{13}$$

where k_N is a thinning density factor determined by the type of thinning. When thinning intensity is known in terms of relative stem volume removed, then the number of stems removed in thinning can be derived from Eq. (13), provided the type of thinning is specified using the thinning density factor k_N.

Additional stand variables

Additional stand variables such as basal area (G) and average diameter-at-breast-height (D_g, in cm) can be derived from total stand volume using an empirical stand form factor that depends on stand height, and thus on site class (Nicke, 1989). Basal area is calculated from $G_t=V_t/F_H$ with G_t representing basal area in m^2 ha^{-1}. D_g is simply calculated from $D_g=200\sqrt{[G/(\pi N)]}$.

Model evaluation

The descriptions of relative volume increment (Eq. (6)) and other growth functions have been analysed statistically by Römisch (1984) by analysing the residuals after fitting the equations to field data using nonlinear regression analysis. This was conducted as part of the studies by Zimmermann (1974), Nake (1983), Römisch (1983) and Gerold (1985). Detailed results of these studies can be found in Gerold (1986) and Wenk et al. (1990). In these studies, it was concluded that the best growth function in general does not exist, but that the choice very much depends on the criteria selected for evaluation, and on the database used for fitting the models. Overall, Eq. (5) performed satisfactorily because of the stability invoked by the parameter c_1. The thinning model was studied by Zimmermann (1974, 1977) and by Wenk and Nicke (1985) by testing it against yield tables and long-term record from permanent plots, and it was concluded that the thinning model can be used, provided thinning intensity does not exceed volume increment during stand development.

Various functions have been tested for evaluation of the height growth model (Römisch, 1979; Nake, 1983; Davids, 1987). Height developments in Norway spruce have been tested intensively, using a large database with permanent plot records. Again, as with volume increment, various functions may be used, depending on the fitting procedure and the database. With regard to the height growth model, the allometrical relation in Eq. (11) can be used equally well (Wenk et al., 1990). In fact, through Eq. (11), height growth is fully determined by the growth parameter c_1 in Eqs. (5) and (6), and the allometric exponent m in Eq. (11).

The description of thinning essentially consists of the amount of volume removed, expressed by the thinning intensity factor k_V (Eq. (7)), and the number of trees removed, with the thinning density factor k_N (Eq. (13)) defining the type of thinning. The intensity parameter k_V is largely independent of site quality and tree species. Its value ranges from 1.2 for light thinning, to 1.45 for moderate thinning, 1.7 for heavy thinning, and 2.0 for very heavy thinning. In general, for an optimal growing condition, it lies between 1.4 and 1.5. The thinning density factor k_N is an effective parameter to describe a wide range of thinning types (Zimmermann, 1974; Nicke, 1989), with $k_N < 1$ resulting in thinning from below, $k_N = 1$ equal or random thinning, and $k_N > 1$ describing thinning from above. In the case $k_N = 0.24$, the thinning comes close to natural self-thinning, with $k_N = 0.33$ then indicating light thinning from below, $k_N = 0.47$ moderate thinning from below, etc. (Halaj, 1975).

Model results

With the model described above, a yield table for Norway spruce was constructed, that is currently being used in forestry practices in the eastern part of Germany (Wenk and Gerold, 1984). The table consists of three different site index classifications, with equal site index (expressed as mean height at 100 years) but with different height growth curves. These differences in height growth correspond to differences in volume increment, and furthermore lead to differences in the timing of maximum mean annual increment (I_m). This allows for an adjustment to growth rates of stands with low site index, for which yield tended to be underestimated in the previously used yield tables (Wiedemann, 1936). Using the model, additional yield tables have been constructed for red oak (*Quercus rubra* L.), common alder (*Alnus glutinosa* (L.) Gaertn.) and northern pine (*Pinus strobus* L.). Summaries of these tables can be found in Wenk et al., 1990).

The model is being updated constantly, and now is available in a practically applicable form (Nicke, 1990; Von Pistor, 1991). Calculations can be performed with constant values for the parameters c_1, m, k_V and k_N, and with parameters varying during the simulation run. The simulation can be carried out with intervals of 5 or 10 years. It has been shown that with varying parameter values, stand development of permanent plots can be reconstructed almost identically (Nicke, 1989). In that case, however, the practical application is severely restricted by the need for detailed specification of the input parameters. In comparison, by application of the model using parameter values that were constant during stand development, yield predictions of acceptable accuracy were obtained using experimental plots for southern Germany (Wenk et al., 1990).

At present, the model is being extended to applications in mixed stands, and the thinning algorithm is made more versatile. For this purpose, stand description is extended with a diameter distribution submodel and a submodel describing merchantable timber in various size classes. Also, the description of volume and height growth is extended to account for long-term effects of environmental changes in site conditions.

Concluding remarks

The treatment of stand development on the basis of a simple growth multiplier as applied here, with the multiplier derived from a description of relative volume increment is straightforward and allows for the application of a wide range of growth functions within a single framework. It can be shown that this approach is useful to fit such growth functions to permanent field plot data, and that stand development can be reproduced satisfactorily. Here, Eq. (5) is used only because the author's experience is mostly with this growth description. The combined treatment of increment and thinning as described here, appears to be a convenient way to model stand development. Here, Eq. (9) was applied for this, but the description of thinning can be replaced by other approaches as well. In particular, the assumption on thinning not exceeding increment since the last thinning may not be fulfilled in many cases. This is clearly the case in mixed stands, where this assumption may be true for the entire stand, but not for individual species in the mixture. Thus, for such applications, the thinning model has to be reconsidered. The combination of volume removal and stem reduction in case of thinning enables systematic treatment of stand development in managed stands with only a limited number of parameters. This approach to thinning simulation can be driven both by the thinning intensity factor as well as by the thinning density factor, depending on the purpose of the model. Because only few parameters are needed, it is relatively easy to formulate the model in a target-mode, in which the final yield determines the required thinning.

The model is suitable for analysing permanent plot data and for constructing yield tables with relatively few parameters that can be interpreted in silvicultural terms. It is therefore expected that the model is of use also when alternative silvicultural treatments have to be evaluated, when e.g. only few field data are available. At present, single tree models are preferred over stand models by many forest scientists that are dealing with stand growth. In addition to this, the trend to complex ecosystem models is evident. Even so, stand-level models will not be ruled out because of their simplicity, general applicability, and reliability. So far, the highly sophisticated growth models that have been developed in the recent past, have not found their way into practical forestry. The model presented in this paper aims to provide practical forestry with a straightforward stand growth model that can be used to deal with issues arising from forest practice.

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